From Modal Logic to Temporal Calculus Models

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The Logic of Time: Modal vs First-Order Approaches

Temporal Logic

Classical logic was not designed for the expression of time and change.

There are two main ways of building temporality into logic:

- The **modal** approach: Extend the logical apparatus with operators expressing temporality.
- The **first-order** approach: Incorporate temporality into non-logical vocabulary.

In the modal approach, time is built into the formal framework in which we express propositions.

In the first-order approach, the formal framework is the same as before, and time is part of the subject-matter, i.e., what we express propositions about.

The Modal Approach: Tense Logic

Temporal operators resemble the *tenses* of natural language:

Formula	Interpretation
p	It is cold
Pp	It was cold, it has been cold
Fp	It will be cold
Hp	It has always been cold
Gp	It will always be cold

Combination of operators:

HFp	It has always been going to be cold
FPp	It will have been cold

An **axiom**:

$p \to GPp$ What is true now will always have been true

An extension of Tense Logic: Hybrid Logic

How can we say more exactly *when* something is true? (I.e., not just past, present, or future.)

Let t stand for the proposition "It is 12th July 2009", and r for "It is raining". Then the formula

$$P(t \wedge r) \vee (t \wedge r) \vee F(t \wedge r)$$

states that it was, is, or will be raining on that day.

This can be abbreviated to

$$\Diamond(t \wedge r)$$

which in Hybrid Logic notation is

$$@_t r.$$

A First-Order Approach: Method of Temporal Arguments

Times are assumed to be individual entities that can be referred to by terms, which in turn can be used as arguments to predicates.

- It rained on 12th July 2009:

$$Rain(day_{12-07-2009})$$

- Napoleon invaded Russia in 1812:

$$Invade(napoleon, russia, year_{1812})$$

Note: This method does not readily distinguish between processes and events. Nor does it specify exactly how the process or event is related to the given time.

Reification

In a reified system, the event or process is expressed by a term, the fact of its occurrence by a predicate. There are two kinds of reification: **type-reification** and **token-reification**.

- Method of temporal arguments:

$$Invade(napoleon, russia, year_{1812})$$

- Type-reification (the event term denotes an event *type*):

$$Occurs(invade(napoleon, russia), year_{1812})$$

- Token-reification (the event term denotes an event *token*):

$$\exists e(Invade(napoleon, russia, e) \wedge Occurs(e, year_{1812})).$$

Exactly what does *Occurs* mean?

In interpreting $Occurs(E, t)$ there is a potential ambiguity:

- Does it mean that t is the exact interval over which E occurred?
- Or does it just mean that E occurred sometime within the interval t?

It is usual to choose the first of these interpretations. This is secured by means of an axiom such as

$$\forall e \forall i \forall i' (Occurs(e, i) \wedge i' \sqsubset i \rightarrow \neg Occurs(e, i'))$$

(here $i' \sqsubset i$ means that i' is a proper subinterval of i).

Given this, the second interpretation can be expressed as

$$\exists i' (i' \sqsubseteq i \wedge Occurs(e, i')).$$

States, Processes, and Events

What happens: States, Processes, and Events

There are many different ways of describing and classifying what goes on in time.

It is common to distinguish three main categories: *states*, *processes*, and *events*.

Each of these characterises a situation from a different point of view:

- A **state** abstracts away from any changes that are taking place and focuses on the unchanging aspects of a situation.
- A **process** focuses on ongoing change as it proceeds from moment to moment, not as a completed whole.
- An **event** is an episode of change with a beginning and an end, considered as a completed whole.

Two kinds of process

"TRUE" PROCESSES *Ongoing open-ended activity*	ROUTINES *Closed sequence of actions leading to definite endpoint*
flowing of river or ocean current	making a pot of tea
back-and-forth movement of tides	baking a cake
growth of a tree	shutting down computer
raining	constructing by-pass
photosynthesis	boarding a plane
coastal erosion	performing appendicectomy
walking, running, eating, singing	giving birth

How do true processes differ from routines?

PROCESSES	ROUTINES
At sufficiently coarse granularity, processes may be conceptualised as homogeneous	Each instantiation of a routine is an event, which at sufficiently coarse granularity may be conceptualised as point-like.
A process can in principle stop at any time without thereby being considered 'incomplete'	There can be incomplete instantiations of a routine, which are interrupted before they finish
A process is like an ordinary object in that it can be meaningfully said to undergo change (e.g., becoming faster or slower)	It does not seem to make sense to ascribe change to routines

"Chunks" of process

A **chunk** of a process is a *bounded instantiation of a process*

A chunk of walking occurs if someone *starts* walking, walks *for a while*, and then *stops* walking.

NOTE: A chunk of walking includes both a beginning and an ending.
A five-minute stretch of walking in the middle of a ten-minute stretch of walking is not a chunk of walking. There are no "subchunks".

Although walking is a process, a chunk of walking is an event.

How do processes and chunks of process differ?

A PROCESS is	A CHUNK OF PROCESS is
open-ended: it does not include start and end points;	**closed**: delimited by starting and stopping events which form essential parts of the chunk;
dissective: any part of a period of running is a period of running;	**non-dissective**: no part of a chunk of running is itself a chunk of running.

Various kinds of event

- **Transitions.** A transition from a situation in which some proposition holds to one in which it does not, or vice versa. Typical examples: the water starts to flow, the sun rises or sets, it starts or stops raining.
- **Chunks of process.** e.g., someone walks, runs, sings, eats, or sleeps for a while, an object falls to the ground, a bird flies from one tree to another.
- **Instantiations of routines.** Specific occurrences consisting of complete or incomplete instantiations of some routine, e.g., someone making a cup of tea, or giving birth, on a particular occasion

Although events may be **punctual** (instantaneous) or **durative** (taking time), there is always some temporal scale (granularity level) at which they can be conceptualised as pointlike.

Relationships between events and processes

Events are dependent on processes in the following ways:

- A durative event is "made of" processes, e.g., *He walked for an hour*, an hour-long event made of walking (cf., a metre-long plank made of wood).
- A durative event may be an instantiation of a complex routine, composed of a number of distinct process chunks representing different phases (cf., a table made of several pieces of wood and metal).
- A punctual event is usually the onset or cessation of a process ("It started raining").

Relationships between processes and events

Processes are dependent on events in the following ways:

- A process may be an open-ended repetition of some event or sequence of events. E.g., the process of hammering consists of a repetition of individual hammer-blows.
- A “higher-level” process may exist by virtue of some complex event (e.g., a routine) being under way, e.g., a house is being built: this takes different forms at different stages, but we can think of what is going on at these different stages as all one process by virtue of its relationship to the completed event.

Types and Tokens

We distinguish between generic *types* and individual *tokens*, i.e., instances, of those types.

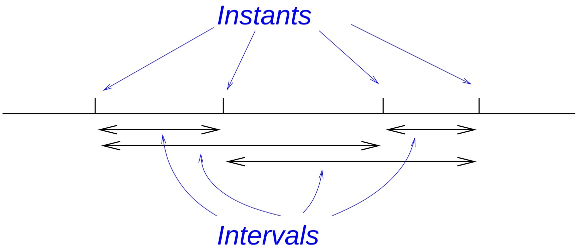
- **Events.** Fairly straightforward:
  - Type: *Earthquake*
  - Tokens: *Lisbon earthquake 1755, San Francisco earthquake 1906, . . .*
- **Processes.** More problematic: What counts as a token of a process?
  - "The rain became heavier". The *same* rain?
  - "The flow of the river stopped in June and began again in September". The same flowing process?

A systematic ontology of processes for use in an information system has to provide consistent answers to questions like this.

Formal Properties of Instants and Intervals

Time Itself: Instants and Intervals

- Instants are durationless. They represent the meeting-points of contiguous intervals. E.g., "2.45 p.m. exactly".
- Intervals have duration. An interval is bounded by instants at the beginning and end. Instants may be
  - "Standard": 1812, June 1812, 24th June 1812.
  - "Arbitrary": from 4 p.m. to 5.30 p.m. on 24th June 1812.
  - Defined by events: The reign of Louis XIV.



Instants and Intervals I

Which is more fundamental, the instant or the interval?

If instants are fundamental, then an interval can be specified by means of its beginning and end points:

$$i = \langle t_1, t_2 \rangle \quad \text{(where } t_1 \prec t_2\text{)}$$

where $x \prec y$ is read 'x precedes y'.

You might (but don't have to) then identify the interval with the *set* of instants falling between the two ends:

$$i = \{t \mid t_1 \prec t \prec t_2\}$$

where $x \prec y \prec z$ is short for $(x \prec y) \wedge (y \prec z)$.

Instants and Intervals II

If intervals are fundamental, then an instant can be specified by means of a pair of intervals:

$$\langle i_1, i_2 \rangle \quad (\text{where } i_1 \,|\, i_2)$$

($x \,|\, y$ is read 'x meets y').

Then we define equality for instants by

$$\langle i_1, i_2 \rangle = \langle j_1, j_2 \rangle =_{\mathrm{def}} i_1 \,|\, j_2 \wedge j_1 \,|\, i_2.$$

In effect, we are defining an instant as an equivalence class of interval-interval pairs.

An Instant-Based Theory of Time

Temporal Precendence

Primitive relation: $t \prec t'$
Interpretation: Instant t precedes (i.e., is earlier than) instant t'.

A **predecessor** of instant t is any instant t' such that $t' \prec t$.
A **successor** of instant t is any instant t' such that $t \prec t'$.

The formal properties of the ordering of the instants are expressed by means of *axioms* written as first-order formulae.

In any application context, the axioms should be chosen to capture the properties of the temporal ordering that are required for reasoning within that context. In principle, different applications may require different models of time (there is not "one true model" for time — probably).

Fundamental Properties of Temporal Precendence

Note: We use the convention that unless otherwise indicated, all individual variables are understood as universally quantified.

- **Irreflexive**:

TI $$\neg(t \prec t)$$

- **Transitive**:

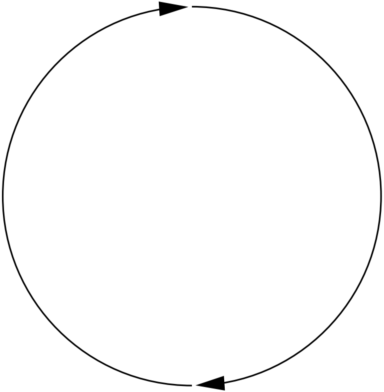
TT $$(t \prec t') \wedge (t' \prec t'') \rightarrow t \prec t''$$

From TI and TT we can infer [Exercise!]

- **Asymmetric**:

TA $$t \prec t' \rightarrow \neg(t' \prec t)$$

The 'flow' of time I: Cyclic Time



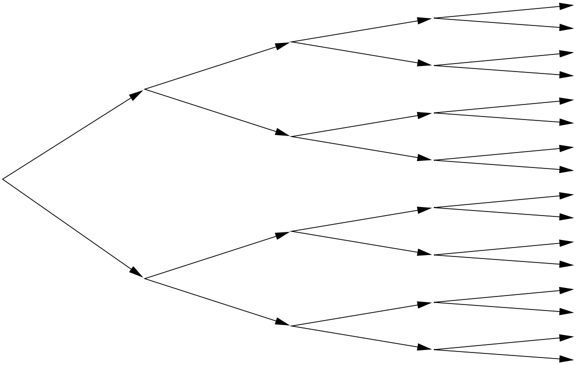
Ruled out by **TA**.

A model for cyclic time:

$$\mathit{Mon} \prec \mathit{Tue} \prec \mathit{Wed} \prec \mathit{Thu} \prec \mathit{Fri} \prec \mathit{Sat} \prec \mathit{Sun} \prec \mathit{Mon}$$

The ‘flow’ of time II: Branching time

Diverging time *branches into the future*:



More than one future for each instant.

Converging time is analogous: more than one past for each instant.

The ‘flow’ of time III: Linearity

- Past-linearity rules out convergence:

TLP $(t' \prec t) \wedge (t'' \prec t) \rightarrow (t' \prec t'') \vee (t'' = t') \vee (t'' \prec t')$

- Future-linearity rules out divergence:

TLF $(t \prec t') \wedge (t \prec t'') \rightarrow (t' \prec t'') \vee (t'' = t') \vee (t'' \prec t')$

The conjunction of **TLP** and **TLF** allows *parallel time lines*:

To rule this out too we need (full) *linearity*:

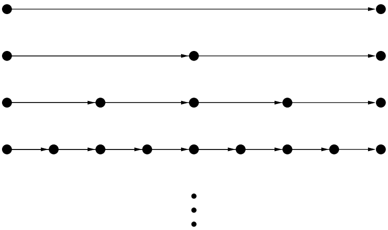
TL $(t \prec t') \vee (t = t') \vee (t' \prec t)$

The 'flow' of time IV: Density

Dense time: Between any two instants there is a third:

TD $$t \prec t' \to \exists t''(t \prec t'' \prec t')$$

Together with TT and TI this implies there are infinitely many times (so long as there are at least two):



This model is presupposed by assigning real or rational numbers to individual instants.

The ‘flow’ of time V: Discreteness

Discrete time: If an instant has a predecessor it has an immediate predecessor, and likewise with successors. (Two axioms)

- Past-discreteness:

TDiP $$t' \prec t \rightarrow \exists t''(t'' \prec t \wedge \neg \exists u(t'' \prec u \prec t))$$

- Future-discreteness:

TDiF $$t \prec t' \rightarrow \exists t''(t \prec t'' \wedge \neg \exists u(t \prec u \prec t''))$$

This model is presupposed by assigning only integers to individual instants.

The ‘flow’ of time VI: Bounding

- Unbounded in the past (no first instant):

TUP $\exists t'(t' \prec t)$

- Unbounded in the future (no last instant):

TUF $\exists t'(t \prec t')$

- Bounded in the past (there is a first instant):

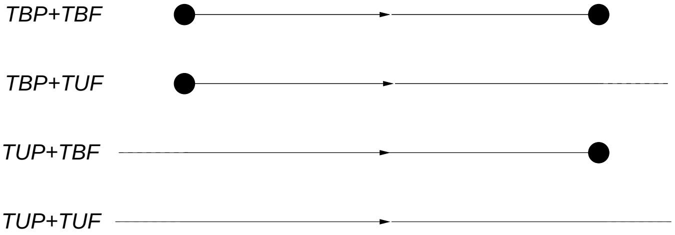
TBP $\exists t \forall t'(t \preceq t')$

- Bounded in the future (there is a last instant):

TBF $\exists t \forall t'(t' \preceq t)$

The ‘flow’ of time VI (contd)

Each of **TBP** and **TUB** can be combined with either **TBF** or **TUF**, giving four possibilities in all:



An Interval-Based Theory of Time

The logic of intervals

James Allen (1984) argued that instants have no empirical reality and that all reasoning about temporal phenomena should be based on a model of time in which intervals are primitive elements, not constructed as aggregates of instants.

He devised a set of 13 basic qualitative relations between intervals, forming a **jointly exhaustive and pairwise disjoint** (JEPD) set.

These can all be defined in terms of a single primitive relation, *meets*, denoted $|$ (or sometimes m), where $a \mid b$ means that interval a ends exactly as interval b begins.

Reference:

James F. Allen, 'Towards a general theory of action and time', *Artificial Intelligence*, **23** (1984) 123–154.

Axioms for 'Meets'

The following is a commonly-used set of axioms for the 'meets' relation $|$:

(M1) $u \,|\, v \wedge u \,|\, w \wedge x \,|\, v \rightarrow x \,|\, w$

(M2) $u \,|\, v \wedge w \,|\, x \rightarrow u \,|\, x \vee \exists y(u \,|\, y \,|\, x) \vee \exists z(w \,|\, z \,|\, v)$

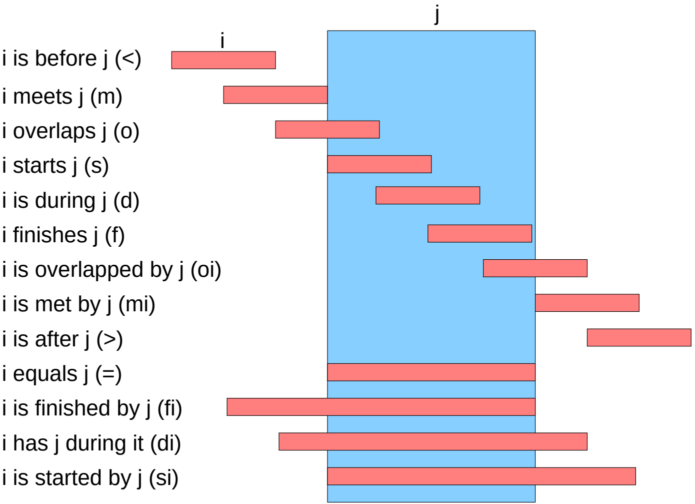
(M3) $\exists v \exists w(v \,|\, u \,|\, w)$

(M4) $u \,|\, v \,|\, x \wedge u \,|\, w \,|\, x \rightarrow v = w$

(M5) $u \,|\, v \rightarrow \exists w \forall x \forall y(x \,|\, u \wedge v \,|\, y \rightarrow x \,|\, w \,|\, y)$

Relations between intervals

The 13 interval–interval relations are illustrated schematically here:



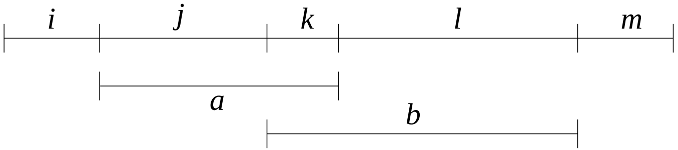
Definition of interval relations in terms of 'meets'

Name	Symbol	Definition
is before	$<$	$a < b \equiv \exists j(a \mid j \mid b)$
meets	$\mid$	Primitive
overlaps	o	$a \,\mathrm{o}\, b \equiv \exists i \exists j \exists k \exists l \exists m(i \mid j \mid k \mid l \mid m \wedge i \mid a \mid l \wedge j \mid b \mid m)$
starts	s	$a \,\mathrm{s}\, b \equiv \exists i \exists j \exists k(i \mid a \mid j \mid k \wedge i \mid b \mid k)$
finishes	f	$a \,\mathrm{f}\, b \equiv \exists i \exists j \exists k(i \mid j \mid a \mid k \wedge i \mid b \mid k)$
is during	d	$a \,\mathrm{d}\, b \equiv \exists i \exists j \exists k \exists l(i \mid j \mid a \mid k \mid l \wedge i \mid b \mid l)$
equals	$=$	$a = b \equiv \exists i \exists j(i \mid a \mid j \wedge i \mid b \mid j)$
is after	$>$	$a > b \equiv b < a$
is met by	mi	$a \,\mathrm{mi}\, b \equiv b \mid a$
is overlapped by	oi	$a \,\mathrm{oi}\, b \equiv b \,\mathrm{o}\, a$
is started by	si	$a \,\mathrm{si}\, b \equiv b \,\mathrm{s}\, a$
is finished by	fi	$a \,\mathrm{fi}\, b \equiv b \,\mathrm{f}\, a$
contains	di	$a \,\mathrm{di}\, b \equiv b \,\mathrm{d}\, a$

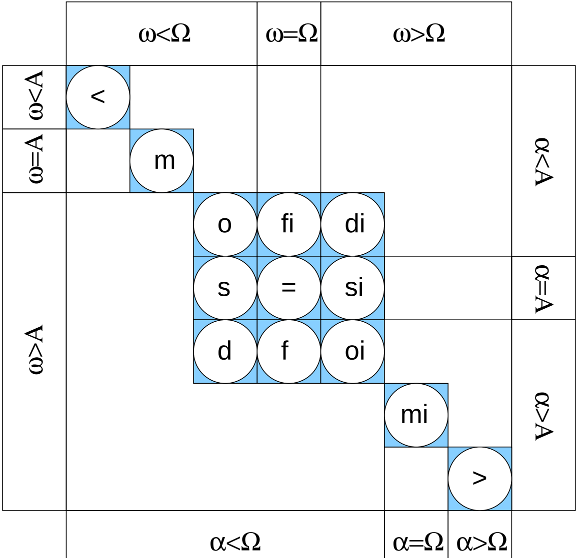
How the definitions work

The following diagram illustrates the definition

$$a \circ b \equiv \exists i \exists j \exists k \exists l \exists m (i \mid j \mid k \mid l \mid m \wedge i \mid a \mid l \wedge j \mid b \mid m)$$



Freksa's Construction: Relations between (α, ω) and (A, Ω)



Conceptual Neighbourhood

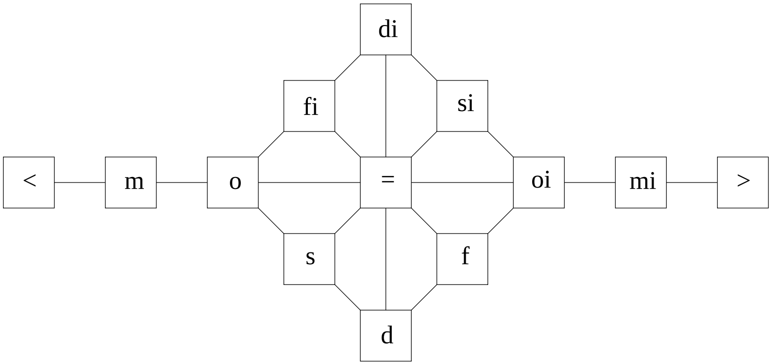
The following definition is due to Freksa (1992):

> *Two relations between pairs of events are (conceptual) neighbours, if they can be directly transformed into one another by continuously deforming (i.e., shortening, lengthening, moving) the events (in a topological sense).*

Freksa's conjecture: "If a cognitive system is uncertain as to which relation between two events holds, uncertainty can be expected particularly between neighbouring concepts."

These ideas can be applied to spatial relations as well as temporal ones (cf., RCC, to be introduced later.).

Conceptual Neighbourhood Diagram



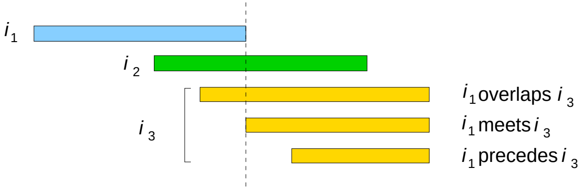
Compositional Reasoning

Composition of interval relations

Given that

The time of the earthquake overlaps the time of the landslide
The time of the landslide overlaps the collapse of the dam

what is the relation between the time of the earthquake and the collapse of the dam?



Conclusion: The time of the earthquake overlaps, meets or precedes the collapse of the dam.

Composition rules

The example on the preceding slide is an example of a **composition rule**.

Composition rules for relations take the form:

- If a stands in relation R to b and b stands in relation S to c, then a stands in one of the relations $T_1, T_2, \ldots, T_n$ to c.

Our example can be written as

$$a \circ b \wedge b \circ c \rightarrow a \circ c \vee a \,|\, c \vee a < c.$$

The **Composition Table** for a set $\mathcal{R}$ of JEPD relations gives the composition rule for every pair of relations $\langle R, S \rangle \in \mathcal{R} \times \mathcal{R}$.

Composition table for the Interval Calculus (part)

	<	m	o	d
<	<	<	<	<
m	<	<	<	o d m
o	<	<	< o m	o d
d	<	< m	< o d m	d

Composition table for the Interval Calculus (part)

	<	m	o	d	…
<	<	<	<	<	
m	<	<	<	o d m	
o	<	<	< o m	o d	
d	<	< m	< o d m	d	
…					

Proving the rule

We can prove the overlap-overlap rule from the axioms for 'meets', and the definition

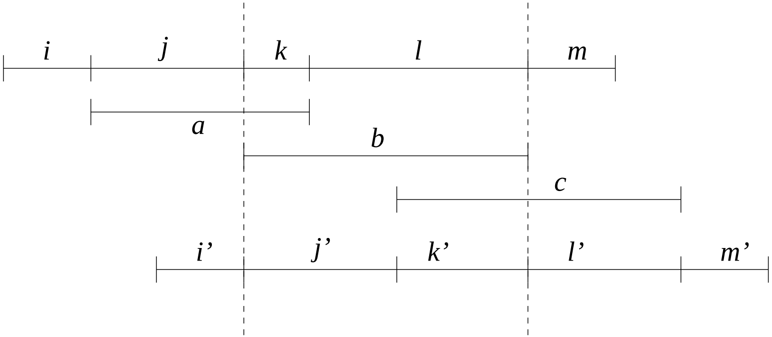
$$a \circ b \equiv \exists i \exists j \exists k \exists l \exists m (i \mid j \mid k \mid l \mid m \wedge i \mid a \mid l \wedge j \mid b \mid m)$$

Given $a \circ b \wedge b \circ c$, this means there exist intervals $i, j, k, l, m, i', j', k', l', m'$ such that

$$\begin{array}{l} i \mid j \mid k \mid l \mid m \ \wedge \\ i \mid a \mid l \ \wedge \\ j \mid b \mid m \ \wedge \\ i' \mid j' \mid k' \mid l' \mid m' \ \wedge \\ i' \mid b \mid l' \ \wedge \\ j' \mid c \mid m' \end{array}$$

This is shown in the diagram on the next slide.

Proving the rule (continued)



The main unknown is the relative ordering of the meeting points of k with l and j' with k'.

Proving the rule (continued)

By axiom M2 we have

$$k \mid k' \vee \exists y(k \mid y \mid k') \vee \exists z(j' \mid z \mid l).$$

- If the first disjunct holds, we have $a \mid l \wedge k \mid l \wedge k \mid k'$, so by axiom M1, we have $a \mid k'$. We then have $a \mid k' \wedge j' \mid k' \wedge j' \mid c$ so by M1 again we have $a \mid c$.
- If the second disjunct holds, we have $a \mid l \wedge k \mid l \wedge k \mid y$, so by M1 we have $a \mid y$. Similarly, from $y \mid k' \wedge j' \mid k' \wedge j' \mid c$ we have $y \mid c$. Hence we have $\exists y(a \mid y \wedge y \mid c)$, which by definition is equivalent to $a < c$.
- The third disjunct is more complicated, but it can be shown that it leads to the result $a \circ c$ (making use of M5 also).

The Interval Algebra

The 13 relations of the Interval Calculus do not form a closed set under composition: in many cases the composition of two relations is a disjunction of two or more relations in the set.

We denote these disjunctions in the form $\{< . \mid, \mathrm{o}, \mathrm{fi}, \mathrm{di}\}$, where

$$a\{<, \mid, \mathrm{o}, \mathrm{fi}, \mathrm{di}\}b \equiv a < b \vee a \mid b \vee a\,\mathrm{o}\,b \vee a\,\mathrm{fi}\,b \vee a\,\mathrm{di}\,b$$

The full set of $2^{13} = 8192$ subsets of the Interval Calculus relations is closed under composition. It is known as the **Interval Algebra**, denoted $\mathcal{A}$.

The composition table for $\mathcal{A}$ has $8192^2 = 67\,108\,864$ entries, which can be readily computed from the 169 entries of the composition table for the Interval Calculus.

Constraint Satisfaction over $\mathcal{A}$

An instance of the **constraint satisfiability problem** over $\mathcal{A}$ consists of a set S of **constraints** each having the form

i stands in relation R to j,

where 'i' and 'j' are variables standing for intervals, and 'R' is one of the relations in $\mathcal{A}$.

Given S, the problem is assign actual intervals (represented by real-number pairs, e.g., (1.53,2.76)) to the variables appearing in S, in such a way that all the constraints in S are satisfied.

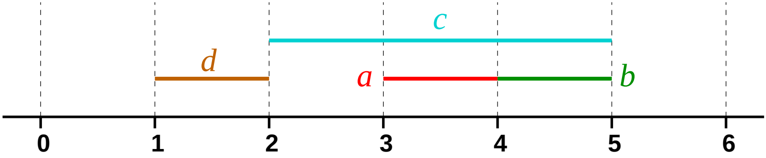
Example of the Constraint Satisfiability Problem

Constraints:

$$a\{\,|\,,\ \mathrm{o}\,\}b, \qquad b\{\,\mathrm{f}\,, =, \ \mathrm{fi}\,\}c, \qquad c\ \mathrm{mi}\ d, \qquad d < a$$

Sample solution:

$$a = (3,4), \quad b = (4,5), \quad c = (2,5), \quad d = (1,2)$$



Computational considerations

- The constraint satisfiability problem for the Interval Algebra is NP-complete (Vilain and Kautz, 1986)
- Assuming P$\neq$NP, this means that temporal reasoning using the full Interval Algebra is intractable (probably of exponential complexity in the worst case).
- Nebel & Bürckert (1995) and Drakengren & Jonsson (1998) identified *maximal tractable subalgebras* of $\mathcal{A}$.
- Krokhin *et al.* (2003) provided a complete enumeration of *all* the tractable subalgebras of $\mathcal{A}$.