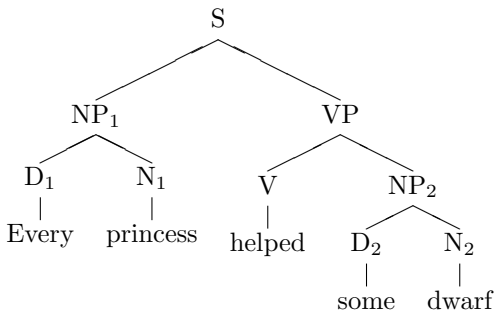
We want to compute the meaning of the sentence "Every princess helped some dwarf". This sentence has the following syntactic structure:

(1)



We have the following basic (non-continuized) meanings:

$$\begin{aligned}
[\![\mathrm{D}_1]\!] = [\![\text{every}]\!] &= \lambda P \lambda Q. \forall x. P(x) \to Q(x) \\
[\![\mathrm{N}_1]\!] = [\![\text{princess}]\!] &= \lambda x.\text{princess}(x) \text{ (or just princess)} \\
[\![\mathrm{V}]\!] = [\![\text{helped}]\!] &= \lambda y \lambda x.\text{help}(x, y) \text{ (or just help)} \\
[\![\mathrm{D}_2]\!] = [\![\text{some}]\!] &= \lambda P \lambda Q. \exists y. P(y) \wedge Q(y) \\
[\![\mathrm{N}_2]\!] = [\![\text{dwarf}]\!] &= \lambda x.\text{dwarf}(x) \text{ (or just dwarf)}
\end{aligned}$$

In addition, we define the following functions:

$$\begin{aligned}
&\text{pure}(u) = \lambda k.k(u)^1 \\
&u <*> v = \lambda k.v(\lambda n.u(\lambda m.k(m(n))))^2 \\
&v >>= u = \lambda c.v(\lambda a.(u(a))(c))
\end{aligned}$$

We then have the following continuized grammar:

$\mathrm{S} \to \mathrm{NP\ VP}$	$\overline{\mathrm{S}} = \overline{\mathrm{VP}} <*> \overline{\mathrm{NP}}$
$\mathrm{VP} \to \mathrm{V\ NP}$	$\overline{\mathrm{VP}} = \overline{\mathrm{V}} <*> \overline{\mathrm{NP}}$
$\mathrm{NP} \to \mathrm{D\ N}$	$\overline{\mathrm{NP}} = \overline{\mathrm{N}} >>= [\![\mathrm{D}]\!]$
$\mathrm{V} \to$ helped	$\overline{\mathrm{V}} = \text{pure}([\![\text{help}]\!])$
$\mathrm{N} \to$ princess	$\overline{\mathrm{N}} = \text{pure}([\![\text{princess}]\!])$
$\mathrm{N} \to$ dwarf	$\overline{\mathrm{N}} = \text{pure}([\![\text{dwarf}]\!])$
$\mathrm{D} \to$ every	$[\![\mathrm{D}]\!] = [\![\text{every}]\!]$
$\mathrm{D} \to$ some	$[\![\mathrm{D}]\!] = [\![\text{some}]\!]$

[1] van Eijck and Unger call this `cpsConst`; alternative names include "unit" and "return"
[2] van Eijck and Unger call this `cpsApply`

First, we have $\overline{\mathrm{N}_2} = \mathrm{pure}([\![\mathrm{dwarf}]\!]) = \lambda k.k(\mathrm{dwarf})$. Then we can compute $\overline{\mathrm{NP}_2} = \overline{\mathrm{N}_2} >>= [\![\mathrm{D}_2]\!]$ as follows:

$$\begin{aligned}
\overline{\mathrm{NP}_2} = \overline{\text{some dwarf}} &= \overline{\mathrm{N}_2} >>= [\![\mathrm{D}_2]\!] \\
&= \lambda c.\overline{\mathrm{N}_2}(\lambda a.([\![\mathrm{D}_2]\!](a))(c)) \\
&= \lambda c.(\lambda k.k(\mathrm{dwarf}))(\lambda a.([\![\mathrm{D}_2]\!](a))(c)) \\
&= \lambda c.(\lambda a.([\![\mathrm{D}_2]\!](a))(c))(\mathrm{dwarf}) \\
&= \lambda c.([\![\mathrm{D}_2]\!](\mathrm{dwarf}))(c) \\
&= \lambda c.((\lambda P \lambda Q.\exists y.P(y) \wedge Q(y))(\mathrm{dwarf}))(c) \\
&= \lambda c.(\lambda Q.\exists y.\mathrm{dwarf}(y) \wedge Q(y))(c) \\
&= \lambda c.\exists y.\mathrm{dwarf}(y) \wedge c(y)
\end{aligned}$$

Then, we have $\overline{\mathrm{V}} = \mathrm{pure}([\![\mathrm{help}]\!]) = \lambda n.n(\mathrm{help})$. Then we can compute $\overline{\mathrm{VP}} = \overline{\mathrm{V}} < * > \overline{\mathrm{NP}_2}$ as follows:

$$\begin{aligned}
\overline{\mathrm{VP}} = \overline{\text{helped some dwarf}} &= \overline{\mathrm{V}} < * > \overline{\mathrm{NP}_2} \\
&= \lambda k.\overline{\mathrm{NP}_2}(\lambda n.\overline{\mathrm{V}}(\lambda m.k(m(n)))) \\
&= \lambda k.(\lambda c.\exists y.\mathrm{dwarf}(y) \wedge c(y))(\lambda n.\overline{\mathrm{V}}(\lambda m.k(m(n)))) \\
&= \lambda k.\exists y.\mathrm{dwarf}(y) \wedge (\lambda n.\overline{\mathrm{V}}(\lambda m.k(m(n))))(y) \\
&= \lambda k.\exists y.\mathrm{dwarf}(y) \wedge \overline{\mathrm{V}}(\lambda m.k(m(y))) \\
&= \lambda k.\exists y.\mathrm{dwarf}(y) \wedge (\lambda n.n(\mathrm{help}))(\lambda m.k(m(y))) \\
&= \lambda k.\exists y.\mathrm{dwarf}(y) \wedge (\lambda m.k(m(y)))(\mathrm{help}) \\
&= \lambda k.\exists y.\mathrm{dwarf}(y) \wedge k(\mathrm{help}(y))
\end{aligned}$$

Similarly to $\overline{\mathrm{N}_2}$, we have $\overline{\mathrm{N}_1} = \mathrm{pure}([\![\mathrm{princess}]\!]) = \lambda k.k(\mathrm{princess})$. In the same way, we can compute $\overline{\mathrm{NP}_1} = \overline{\mathrm{N}_1} >>= [\![\mathrm{D}_1]\!]$ as follows:

$$\begin{aligned}
\overline{\mathrm{NP}_1} = \overline{\text{every princess}} &= \overline{\mathrm{N}_1} >>= [\![\mathrm{D}_1]\!] \\
&= \lambda c.\overline{\mathrm{N}_1}(\lambda a.([\![\mathrm{D}_1]\!](a))(c)) \\
&= \lambda c.(\lambda k.k(\mathrm{princess}))(\lambda a.([\![\mathrm{D}_1]\!](a))(c)) \\
&= \lambda c.(\lambda a.([\![\mathrm{D}_1]\!](a))(c))(\mathrm{princess}) \\
&= \lambda c.([\![\mathrm{D}_1]\!](\mathrm{princess}))(c) \\
&= \lambda c.((\lambda P \lambda Q.\forall x.P(x) \to Q(x))(\mathrm{princess}))(c) \\
&= \lambda c.(\lambda Q.\forall x.\mathrm{princess}(x) \to Q(x))(c) \\
&= \lambda c.\forall x.\mathrm{princess}(x) \to c(x)
\end{aligned}$$

Finally, we can compute $\overline{\mathrm{S}} = \overline{\mathrm{VP}} < * > \overline{\mathrm{NP}_1}$ as follows:

$$\begin{aligned}
\overline{\mathrm{S}} = \overline{\text{every princess helped some dwarf}} &= \overline{\mathrm{VP}_1} < * > \overline{\mathrm{NP}_1} \\
&= \lambda k.\overline{\mathrm{NP}_1}(\lambda n.\overline{\mathrm{VP}}(\lambda m.k(m(n)))) \\
&= \lambda k.(\lambda c.\forall x.\mathrm{princess}(x) \to c(x))(\lambda n.\overline{\mathrm{VP}}(\lambda m.k(m(n)))) \\
&= \lambda k.\forall x.\mathrm{princess}(x) \to (\lambda n.\overline{\mathrm{VP}}(\lambda m.k(m(n))))(x) \\
&= \lambda k.\forall x.\mathrm{princess}(x) \to \overline{\mathrm{VP}}(\lambda m.k(m(x))) \\
&= \lambda k.\forall x.\mathrm{princess}(x) \to (\lambda c.\exists y.\mathrm{dwarf}(y) \wedge c(\mathrm{help}(y)))(\lambda m.k(m(x))) \\
&= \lambda k.\forall x.\mathrm{princess}(x) \to \exists y.\mathrm{dwarf}(y) \wedge (\lambda m.k(m(x)))(\mathrm{help}(y)) \\
&= \lambda k.\forall x.\mathrm{princess}(x) \to \exists y.\mathrm{dwarf}(y) \wedge k((\mathrm{help}(y))(x)) \\
&= \lambda k.\forall x.\mathrm{princess}(x) \to \exists y.\mathrm{dwarf}(y) \wedge k(\mathrm{help}(x,y))
\end{aligned}$$

To get the meaning of S, we apply $\overline{\mathrm{S}}$ to the trivial continuation $\lambda x.x$:

$$\begin{aligned}
[\![\mathrm{S}]\!] = [\![\text{every princess helped some dwarf}]\!] &= \overline{S}(\lambda x.x) \\
&= (\lambda k.\forall x.\mathrm{princess}(x) \to \exists y.\mathrm{dwarf}(y) \wedge k(\mathrm{help}(x,y)))(\lambda x.x) \\
&= \forall x.\mathrm{princess}(x) \to \exists y.\mathrm{dwarf}(y) \wedge (\lambda x.x)(\mathrm{help}(x,y)) \\
&= \forall x.\mathrm{princess}(x) \to \exists y.\mathrm{dwarf}(y) \wedge \mathrm{help}(x,y)
\end{aligned}$$

Note that with an alternative definition of $u < * >' v = \lambda k.u(\lambda m.v(\lambda n.k(m(n))))$[3], we can derive the reverse scope. The derivation is left as an exercise for the reader[4].

[3] van Eijck and Unger call this `cpsApply'`

[4] Specifically, Exercise 11.6