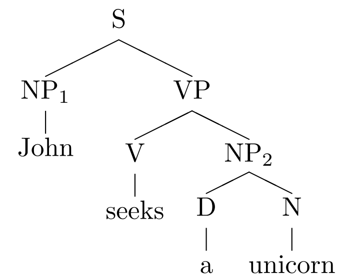
We want to compute the meaning of the sentence "John seeks a unicorn". This sentence has the following syntactic structure:

(1)




We have the following basic meanings:

$$\begin{aligned}
\llbracket \mathrm{NP}_1 \rrbracket &= \llbracket \mathrm{John} \rrbracket &&= \lambda P \lambda w.P(j)(w)\\
\llbracket \mathrm{V} \rrbracket &= \llbracket \mathrm{seeks} \rrbracket &&= \lambda \mathcal{P} \lambda y \lambda w''.\forall w'.\mathrm{try_to}(w'', w', y(w'')) \to \mathcal{P}(\lambda z \lambda w'''.\mathrm{find}(w''', y(w'''), z(w''')))(w')\\
\llbracket \mathrm{D} \rrbracket &= \llbracket \mathrm{a} \rrbracket &&= \lambda P \lambda Q \lambda w.\exists x.P(x)(w) \wedge Q(x)(w)\\
\llbracket \mathrm{N} \rrbracket &= \llbracket \mathrm{unicorn} \rrbracket &&= \lambda y \lambda w'.\mathrm{unicorn}(w', y(w'))
\end{aligned}$$

We compute $\llbracket \mathrm{NP}_2 \rrbracket$ by applying $\llbracket \mathrm{N} \rrbracket$ to $\llbracket \mathrm{D} \rrbracket$:

$$\begin{aligned}
\llbracket \mathrm{NP}_2 \rrbracket = \llbracket \text{a unicorn} \rrbracket &= (\lambda P \lambda Q \lambda w.\exists x.P(x)(w) \wedge Q(x)(w))(\lambda y \lambda w'.\mathrm{unicorn}(w', y(w')))\\
&= \lambda Q \lambda w.\exists x.(\lambda y \lambda w'.\mathrm{unicorn}(w', y(w')))(x)(w) \wedge Q(x)(w)\\
&= \lambda Q \lambda w.\exists x.(\lambda w'.\mathrm{unicorn}(w', x(w')))(w) \wedge Q(x)(w)\\
&= \lambda Q \lambda w.\exists x.\mathrm{unicorn}(w, x(w)) \wedge Q(x)(w)
\end{aligned}$$

Now we apply $\llbracket \mathrm{NP}_2 \rrbracket$ to $\llbracket \mathrm{V} \rrbracket$ to get $\llbracket \mathrm{VP} \rrbracket$:

$$\begin{aligned}
\llbracket \mathrm{VP} \rrbracket = \llbracket \text{seeks a unicorn} \rrbracket &= (\lambda \mathcal{P} \lambda y \lambda w''.\forall w'.\mathrm{try_to}(w'', w', y(w'')) \to \mathcal{P}(\lambda z \lambda w'''.\mathrm{find}(w''', y(w'''), z(w''')))(w'))(\lambda Q \lambda w.\exists x.\mathrm{unicorn}(w, x(w)) \wedge Q(x)(w))\\
&= \lambda y \lambda w''.\forall w'.\mathrm{try_to}(w'', w', y(w'')) \to (\lambda Q \lambda w.\exists x.\mathrm{unicorn}(w, x(w)) \wedge Q(x)(w))(\lambda z \lambda w'''.\mathrm{find}(w''', y(w'''), z(w''')))(w')\\
&= \lambda y \lambda w''.\forall w'.\mathrm{try_to}(w'', w', y(w'')) \to (\lambda w.\exists x.\mathrm{unicorn}(w, x(w)) \wedge (\lambda z \lambda w'''.\mathrm{find}(w''', y(w'''), z(w''')))(x)(w))(w')\\
&= \lambda y \lambda w''.\forall w'.\mathrm{try_to}(w'', w', y(w'')) \to (\lambda w.\exists x.\mathrm{unicorn}(w, x(w)) \wedge (\lambda w'''.\mathrm{find}(w''', y(w'''), x(w''')))(w))(w')\\
&= \lambda y \lambda w''.\forall w'.\mathrm{try_to}(w'', w', y(w'')) \to (\lambda w.\exists x.\mathrm{unicorn}(w, x(w)) \wedge \mathrm{find}(w, y(w), x(w)))(w')\\
&= \lambda y \lambda w''.\forall w'.\mathrm{try_to}(w'', w', y(w'')) \to \exists x.\mathrm{unicorn}(w', x(w')) \wedge \mathrm{find}(w', y(w'), x(w'))
\end{aligned}$$

Finally, we can apply $[\![\text{VP}]\!]$ to $[\![\text{NP}_1]\!]$ to get $[\![\text{S}]\!]$:

$$
\begin{aligned}
[\![\text{S}]\!] = [\![\text{John seeks a unicorn}]\!] &= (\lambda P \lambda w.P(j)(w))(\lambda y \lambda w''.\forall w'.\text{try\_to}(w'', w', y(w'')) \to \exists x.\text{unicorn}(w', x(w')) \wedge \text{find}(w', y(w'), x(w'))) \\
&= \lambda w.(\lambda y \lambda w''.\forall w'.\text{try\_to}(w'', w', y(w'')) \to \exists x.\text{unicorn}(w', x(w')) \wedge \text{find}(w', y(w'), x(w')))(j)(w) \\
&= \lambda w.(\lambda w''.\forall w'.\text{try\_to}(w'', w', j(w'')) \to \exists x.\text{unicorn}(w', x(w')) \wedge \text{find}(w', j(w'), x(w')))(w) \\
&= \lambda w.\forall w'.\text{try\_to}(w, w', j(w)) \to \exists x.\text{unicorn}(w', x(w')) \wedge \text{find}(w', j(w'), x(w'))
\end{aligned}
$$