Anaphora and Ambiguity in Narratives

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A Narrative

The family of Dashwood had long been settled in Sussex. Their estate was large, and their residence was at Norland Park, in the centre of their property, where, for many generations, they had lived in so respectable a manner as to engage the general good opinion of their surrounding acquaintance. The late owner of this estate was a single man, who lived to a very advanced age, and who for many years of his life, had a constant companion and housekeeper in his sister. But her death, which happened ten years before his own, produced a great alteration in his home; for to supply her loss, he invited and received into his house the family of his nephew Mr. Henry Dashwood, the legal inheritor of the Norland estate, and the person to whom he intended to bequeath it.

(Jane Austen, Sense and Sensibility)

Not A Narrative

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(2) Romeo cried. Juliet left.

More than the sums of their parts

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(6) a. B: I burned my dinner.
    b. B: I made steak.

Like words compose to sentences, sentences compose to narratives.

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What are the parts?

And how do they combine?

And to what?

Anaphora

- What about sentences like this:

(7) He walks.

(8) Then someone walked.

(9) So am I.

- Do you know what is required for these sentences to be true?

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such sentences are parts of bigger wholes

Anaphora

- An expression whose meaning depends on a prior expression is called an anaphor.

  (roughly)

(10) There is a man. He walks.

(11) Nobody was moving. Later, someone walked.

(12) Damaya is upset. So am I.

Incoherence

- Call a narrative incoherent if you cannot understand it.
  - More precise definitions of “incoherence” in due time.

- There are different reasons for a sentence to be incoherent, but we will focus first on incoherent use of anaphora.

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(13) There is nobody. ✗He walks.

There are a lot of them!

- We saw pronominal anaphora ("he"), temporal anaphora ("later"), adjectival anaphora ("so").

- Event anaphora:

(14) Tonkee hit Binof. It caused a fight.

- Propositional anaphora:

(15) Damaya believes it is raining. Essun doubts that.

- Type anaphora:

(16) Hoa gave a presentation. Jija gave one too.

… and they are not trivial

- You may now think:

  Say I have two sentences. I understand the truth-conditions of the first, but the second contains a "he". If the truth-conditions of the first are such that there is a male person in every situation where the sentence is true, then "he" refers to this person.

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- You may now think:

  Say I have two sentences. I understand the truth-conditions of the first, but the second contains a "he". If the truth-conditions of the first are such that there is a male person in every situation where the sentence is true, then "he" refers to this person.

- If you think that, you are very very clever!

Not just truth-conditions

- But wrong.

(17) I have three siblings, two of whom are female.
My sisters are here. ✗He is somewhere else.

Not just truth-conditions

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- These are Partee sentences (for the great Barbara Partee).
- This is her example:

(18) Nine of my ten marbles are in the bag.
✗It must be under the sofa.

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- These are Partee sentences (for the great Barbara Partee).
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- The Partee observation is *universal* to anaphora.
  > Try to find your own examples for other cases!

More like this

(19) There are some men. They walk.

- Double negation:

(20) There aren’t no men. ✗They walk.

- Quantifier duality:

(21) It is not the case that everyone is not a man. ✗They walk.

There is only one conclusion to draw:

The referents that an anaphor refers back to are not (merely) part of what is true, but instead they are tied to particular linguistic expressions.

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a. Human: What do you want?
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c. Human: When do you want it?
d. Computer: When do I want what?

Some terminology

- Let's say that an anaphor binds to a previous expression.

(23) A woman is in the park. She walks.

- Let's say that such expressions that anaphora can bind to have binding potential.

Blocked potential

- “someone” has binding potential for pronominal anaphora.

(24) Someone walks. She looks happy.

- But not in all sentences:

(25) It is not the case that someone walks. ✗She looks happy.

(26) Either someone walks or it rains. ✗She looks happy.

(27) If someone walks, it is sunny. ✗She looks happy.

We want a systematic theory

of what binding potential is

and of when we can access this potential.

Binding

Sentential Connectives

- We already know some expressions that compose sentences.
- *and* $\wedge$
- *or* $\vee$
- *if ... then* $\rightarrow$

- And some expressions that modify sentences.
- *not* $\neg$
- *maybe* $\Diamond$

Sentential Connectives (contd.)

- If you have two sentences *A* and *B* which you understand, then you also understand:
- *A and B* is true if *A* is true and *B* is true.
- *A or B* is true if *A* is true or *B* is true.
- *if A then B* is true if *A* is false or *B* is true.
- *not A* is true if *A* is false.
- (let's not worry about *maybe* right now)

A first attempt

- A logician would say that expressions with binding potential are like existential quantifiers.

$$\textit{someone} \approx \exists x$$

(28) Someone walks. She looks happy.

$$\exists x.\mathtt{walk}(x) \;\wedge\; \mathtt{looks\text{-}happy}(x)$$

- Universal quantification does not have binding potential for singular pronominal anaphora.

(29) Everyone walks. She is happy.

$$\forall x.\mathtt{walk}(x) \;\wedge\; \mathtt{looks\text{-}happy}(x)$$

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It really is the anaphor’s fault

- It’s not because there are two anaphora:

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- It's not because there are two anaphora:

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- It's not because "a farmer" is different from "someone":

(32) If someone loves something, he won't beat it.

- It's not because of the "if... then":

(33) Every farmer who owns a donkey beats it.

- Again, this is a *general* property of anaphora.

Adjectival:

(34) If Hoa is away, then so is Damaya.
(They are always together)

Temporal:

(35) If I drink, then I'm hungover the next morning.

Propositional:

(36) If Damaya says something, Essun will question it.

Discourse Representation Theory

Discourse and Story

- In literary criticism, one separates a narrative into story and discourse.
- A discourse is a text. What happens might be reported out of order.
- A story is the sequence of happenings that is described in the text.
- If we are reading a discourse and we cannot determine the story, we find the discourse incoherent.

What is a story?

- What's in a story? Think of it like a theatre play.

1. The referents. (or *dramatis personae*).
2. The conditions: what the referents do / what happens to the referents.

- The sentence "He beats it" does not have truth-conditions.

- It only has meaning if we know which actor "he" is.

Discourse is HOW a narrative is told.

Story is WHAT happens in the narrative.

A story contains things we talk about and what happens to these things.

- So let’s just do **exactly that**.

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- We use a universal language to describe stories called discourse representation theory (DRT).
- Stories contain actors, and say something about these actors.
- $\boxed{\text{Natural Language}\ \textit{Discourses}} \rightsquigarrow \boxed{\textit{Stories}\ \boxed{\frac{\text{actors}}{\text{events}}}} \mapsto \boxed{\text{Truth Conditions}\ \textit{Meaning}}$

Discourse Representation Structures

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- So we construct an intermediate representation for stories.
- A "box" that keeps track of *what there is* separately of *what these things do* is called a Discourse Representation Structure.

$x_1, x_2, \ldots, x_n$	$\leftarrow$ the things we talk about
φ_1 φ_2 $\vdots$ φ_n	$\leftarrow$ what we say about these things

Separation of Reference and Predication

- I think it is an extremely good idea to do it like this.
- We won't do dialogues just now, but consider this:

(37) a. Hoa: There is a cat outside.

b. Jija: No, it's a dog.

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b. Jija: No, it's a dog.

(38) a. Hoa: There is a cat outside.
b. Jija: It is not the case that there is a cat outside. It's a dog.

(39) a. Hoa: There is a cat outside.
b. Jija: There is no cat outside. It's a dog.

A Simple DRS

- "A farmer beats a donkey."
- $$\begin{array}{|l|}\hline f, d \\ \hline \texttt{farmer}(f) \\ \texttt{donkey}(d) \\ \texttt{beat}(f, d) \\ \hline \end{array}$$
- $\left\langle \{f, d\}, \{\texttt{farmer}(f), \texttt{donkey}(d), \texttt{beat}(f, d)\} \right\rangle.$

Now, the stroke of genius (Hans Kamp):

Stories have Sub-stories.

Boxes can appear in boxes.

A Less Simple DRS

(40) I'm having a party.
If Damaya is coming to it, she will bring wine.

$$\begin{array}{|l|}\hline j, p \\ \hline \text{Julian}(j) \\ \texttt{party}(p) \\ \texttt{have}(j, p) \\ \begin{array}{|l|}\hline d \\ \hline \text{Damaya}(d) \\ \texttt{coming}(d, p) \\ \hline\end{array} \Rightarrow \begin{array}{|l|}\hline w \\ \hline \texttt{wine}(w) \\ \texttt{bring}(d, w) \\ \hline\end{array} \\ \hline\end{array}$$

The Language of DRSs

- Take REF a set of variables.
- Take NAME a set of designators ("names").
- Take PRED a set of predicates / properties.
- A Discourse Representation Structure is a box $K = \boxed{\dfrac{\text{REFs}}{\text{CONs}}}$

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- CONs are constructed as follows:
- if N is a NAME and x is a REF, $N(x)$ is a CON;
- if P is a PRED and $x_1, \ldots, x_n$ are REFs, $P(x_1, \ldots, x_n)$ is a CON;
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- if x and y are REFs, $x = y$ is a CON;
- if K is a DRS, then $\neg K$ and $\Diamond K$ are CONs;
- if K and K' are DRSs, then $K \bigvee K'$ is a CON;
- if K and K' are DRSs, then $K \Rightarrow K'$ is a CON.

A new way to think About *if* and *every*

- The idea:
- “If a farmer owns a donkey, he beats it” is a bit like Whenever a farmer owns a donkey, he beats it.
- Better yet, write *Whenever the story is such that it contains a farmer, a donkey and the farmer owns the donkey, then the story is such that the farmer beats the donkey.*

Indefinites vs Quantifiers

"A man walks."

$$\begin{array}{|l|}\hline x \\ \hline \mathtt{man}(x) \\ \mathtt{walks}(x) \\ \hline \end{array}$$

"Every man walks."

$$\begin{array}{|c|}\hline \\ \hline \begin{array}{|l|}\hline x \\ \hline \mathtt{man}(x) \\ \hline \end{array} \Rightarrow \begin{array}{|l|}\hline \\ \hline \mathtt{walk}(x) \\ \hline \end{array} \\ \hline \end{array}$$

Binding Potential

- The conditions in a box can talk about the referents on the top of the same box.
- But sometimes, referents on top of one box are available to talk about in other boxes.
- Intuitively, in a sub-story you can talk about the actors of the bigger story.
- But in the bigger story you are not (always) allowed to speak about actors of a sub-story.

Binding: The Rules (informal)

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\Rightarrow
\begin{array}{|l|}
\hline
w \\
\hline
\texttt{wine}(w) \\
\texttt{bring}(d, w) \\
\hline
\end{array} \\
\hline
\end{array}
$$

And Formally (not as bad as it looks)

Subordination

A DRS K' is **immediately subordinate** to a DRS K iff:

1. K contains the condition $\neg K'$ or $\Diamond K'$; or
2. K contains a condition of the form $K' \vee K''$ or $K'' \vee K'$.
3. K contains a condition of the form $K' \Rightarrow K''$.
4. There is a condition $K \Rightarrow K'$ in some K''.

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K' is **subordinate** to K if K' is connected to K via immediate subordination ("up or left in conditionals").

That is, if there is a chain $K' = K_1, K_2, ..., K_{n-1}, K_n = K$ where for all i, K_i is immediately subordinate to K_{i-1}.

- Now, a pronoun in K' can access referents in all DRSs K that K' is subordinate to.

- Negation blocks binding.

(41) It is not the case that a man is running. #He takes his time.

$$\begin{array}{|l|}\hline \\ \hline \neg \begin{array}{|l|}\hline x \\ \hline \mathtt{man}(x) \\ \mathtt{runs}(x) \\ \hline \end{array} \\ \mathtt{take\_time}(^{?}x) \\ \hline \end{array}$$

Accessibility (Negation)

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\hline
\\
\hline
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x \\
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\hline
\end{array} \\
\mathtt{take\_time}({}^{?}x) \\
\hline
\end{array}
$$

- This is actually fine, but for different reasons (later!):

(42) It is not the case that Jija is running. He takes his time.

Accessibility (Negation)

- Negation blocks binding—but only if the referent is below the negation.

(43) A man is not running. He takes his time.

$$
\begin{array}{|l|}
\hline
x \\
\hline
\texttt{man}(x) \\
\neg \begin{array}{|l|} \hline \\ \hline \texttt{runs}(x) \\ \hline \end{array} \\
\texttt{take\_time}(x) \\
\hline
\end{array}
$$

(44) Not every man is running. #He takes his time.

$$\begin{array}{|l|}\hline \\ \hline \neg \begin{array}{|l|}\hline \\ \hline \begin{array}{|l|}\hline x \\ \hline \mathtt{man}(x) \\ \hline \end{array} \Rightarrow \begin{array}{|l|}\hline \\ \hline \mathtt{runs}(x) \\ \hline \end{array} \\ \hline \end{array} \\ \mathtt{take\_time}({}^{?}x) \\ \hline \end{array}$$

Accessibility (Disjunction)

- Can’t go left or right in disjunction (this is actually controversial).

(45) Either a man is having tea or ${}^{?}$he is having coffee.

$$\begin{array}{|c|}\hline \\ \hline \begin{array}{|l|}\hline x, y \\ \hline \texttt{man}(x) \\ \texttt{tea}(y) \\ \texttt{have}(x, z) \\ \hline \end{array} \bigvee \begin{array}{|l|}\hline z \\ \hline \texttt{coffee}(z) \\ \texttt{have}({}^{?}x, z) \\ \hline \end{array} \\ \hline \end{array}$$

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- This is actually fine, but for different reasons (later!):

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Truth

Interpretation of DRSs: World Model

- We want to have a mathematical notion of truth conditions.

- A model is a tuple $M = (W_M, D_M, N_M, P_M)$ where

- W_M is a set of possible worlds,
- D_M is a set of things (the domain of reference),
- N_M is an assignment of names to things (N_M : NAME $\rightarrow D_M$),
- and P_M is an assignment of properties to the set of all things that have that property in a world ($P_M : W \times \text{PROP} \rightarrow \mathcal{P}(D_M^{<\omega})$.

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- A sentence like "Julian is happy" is true in M, w iff the set $P_M(w, \texttt{happy})$ contains the thing $N_M(\text{Julian})$.
- We write $M, w \models \varphi$ for "φ is true in w according to M".

Interpretation of DRSs: Referent Extension

- The idea is this: a DRSs tells us a story about how some things have some properties.
- To evaluate whether it is true, we need to find people in a world model that have these properties.
- New DRSs may add new people to talk about.

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Referent Assignments

Let $M = (W_M, D_M, N_M, P_M)$ be a model. Let $f, g : \text{REF} \to D_M$ be partial functions from variables to objects in the model.
Write $g > f$ ("g extends f") if $\text{dom}(g) \supseteq \text{dom}(f)$ and for all $x \in \text{dom}(f), f(x) = g(x)$.

Interpretation of DRSs: Truth

- Referents extend variable assignments.
- The conditions impose tests on assignments.

DRT Semantics

Let M be a model. Define by simultaneous recursion for any $w \in W_M$ and any assignments f, g:

1. $f[\![\langle U, Cons\rangle]\!]_{M,w}g$ iff $g > f$, $U \subseteq \mathrm{dom}(g)$ and $M, w, g \models_{DRT} C$ for all $C \in Cons$.

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2. $M, w, f \models_{DRT} R(x_1, \cdots, x_n)$ iff $M, w, f \models R(f(x_1), \cdots, f(x_n))$.

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2. $M, w, f \models_{DRT} R(x_1, \cdots, x_n)$ iff $M, w, f \models R(f(x_1), \cdots, f(x_n))$.
3. $M, w, f \models_{DRT} \neg K$ iff there is no g with $f[\![K]\!]_{M,w}g$.

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1. $f[\![\langle U, Cons\rangle]\!]_{M,w}g$ iff $g > f$, $U \subseteq \mathrm{dom}(g)$ and $M, w, g \models_{DRT} C$ for all $C \in Cons$.
2. $M, w, f \models_{DRT} R(x_1, \cdots, x_n)$ iff $M, w, f \models R(f(x_1), \cdots, f(x_n))$.
3. $M, w, f \models_{DRT} \neg K$ iff there is no g with $f[\![K]\!]_{M,w}g$.
4. $M, w, f \models_{DRT} \Diamond K$ iff there is a $v \in W$ and a g with $f[\![K]\!]_{M,v}g$.

Interpretation of DRSs: Truth

- Referents extend variable assignments.
- The conditions impose tests on assignments.

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5. $M, w, f \models_{DRT} K_1 \bigvee K_2$ iff there is a g with $f[\![K_1]\!]_{M,w} g$ or $f[\![K_2]\!]_{M,w} g$.

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6. $M, w, f \models_{DRT} K_1 \Rightarrow K_2$ iff for every g with $f[\![K_1]\!]_{M,w}g$, there is a h with $g[\![K_2]\!]_{M,w}h$.

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DRT embeds into Modal Predicate Logic (S5)

Define recursively:

- $$\begin{array}{lcl} (P^k(x_1, \ldots, x_k))^\heartsuit & = & P^k x_1, \ldots, x_k; \\ (x_i = x_j)^\heartsuit & = & (x_i = x_j); \\ (\neg K)^\heartsuit & = & \neg K^\heartsuit; \\ (K_1 \vee K_2)^\heartsuit & = & (K_1^\heartsuit \vee K_2^\heartsuit); \end{array}$$

- If $K_1 = \langle \{x_1, \ldots, x_n\}, \{\text{Con}_1, \ldots, \text{Con}_m\} \rangle$, then

$$\begin{array}{ll} K_1^\heartsuit & = \exists x_1 \ldots \exists x_n (\text{Con}_1^\heartsuit \wedge \ \ldots \ \wedge \text{Con}_m^\heartsuit); \\ (K_1 \Rightarrow K_2)^\heartsuit & = \forall x_1 \ldots \forall x_n ((\text{Con}_1^\heartsuit \wedge \ \ldots \ \wedge \text{Con}_m^\heartsuit) \to K_2^\heartsuit). \end{array}$$

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- $M, w, f \models K$ iff $M, w, f \models_{MPL} K^\heartsuit$.

- “If a farmer owns a donkey, he beats it.”
- $$\begin{array}{|c|}\hline \\ \hline \begin{array}{|l|}\hline f, d \\ \hline \texttt{farmer}(f) \\ \texttt{donkey}(d) \\ \texttt{owns}(f, d) \\ \hline \end{array} \Rightarrow \begin{array}{|l|}\hline \\ \hline \texttt{beat}(f, d) \\ \hline \end{array} \\ \hline \end{array}$$
- $\approx$ Whenever we have a farmer and we have a donkey and the farmer owns the donkey, then the farmer beats the donkey.

- $\boxed{\begin{array}{l} \\ \hline \boxed{\begin{array}{l} f, d \\ \hline \mathtt{farmer}(f) \\ \mathtt{donkey}(d) \\ \mathtt{owns}(f, d) \end{array}} \Rightarrow \boxed{\begin{array}{l} \\ \hline \mathtt{beat}(f, d) \end{array}} \end{array}}$ is true for $M, w, \emptyset$ iff

- For every $g > \emptyset$ with $M, w, g \models \boxed{\begin{array}{l} f, d \\ \hline \mathtt{farmer}(f) \\ \mathtt{donkey}(d) \\ \mathtt{owns}(f, d) \end{array}}$

  There is a $h > g$ with $M, w, h \models \boxed{\begin{array}{l} \\ \hline \mathtt{beat}(f, d) \end{array}}$

Donkey Sentences, Formally

- $\begin{array}{|l|}\hline \\ \hline \begin{array}{|l|}\hline f,d \\ \hline \mathtt{farmer}(f) \\ \mathtt{donkey}(d) \\ \mathtt{owns}(f,d) \\ \hline\end{array} \Rightarrow \begin{array}{|l|}\hline \\ \hline \mathtt{beat}(f,d) \\ \hline\end{array} \\ \hline\end{array}$ is true for $M, w, \emptyset$ iff
- Because the top part of the right box is empty, $h = g$.

For every g with $M, g \models \begin{array}{|l|}\hline f,d \\ \hline \mathtt{farmer}(f) \\ \mathtt{donkey}(d) \\ \mathtt{owns}(f,d) \\ \hline\end{array}$, $M, g \models \begin{array}{|l|}\hline \\ \hline \mathtt{beat}(f,d) \\ \hline\end{array}$

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This is true exactly if (in *M*) all farmers beat all their donkeys!

Constructing DRSs

Recap

- "To understand (the meaning of) a sentence is to know the circumstances in which it is true."
- We have seen some sentences that we cannot seem to understand.
  - > incoherent sentences.

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Recap

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- So now we speak not of the truth of sentences (should not do that anyway!) but of the truth of semantic representations.
- "To understand (the meaning of) a sentence is to know the circumstances in which its semantic representation is true."
- $\boxed{\text{Natural Language Sentences}} \rightsquigarrow \boxed{\text{DRSs}} \mapsto \boxed{\text{Models}}$, where:

  $\rightsquigarrow :=$ the discourse representation *construction algorithm*,
  $\mapsto :=$ a truth-conditional model-theoretic *embedding*.

DRS Construction Algorithm (Kamp and Reyle 1993)

DRS-Construction Algorithm

Input: a discourse $D = S_1, ..., S_i, S_{i+1}, ..., S_n$
the empty DRS K_0

Keep repeating for $i = 1, ..., n$**:**

(i) add the syntactic analysis $[S_i]$ of (the next) sentence S_i to the conditions of K_{i-1}; call this DRS K_i*. Go to (ii).

(ii) Input: a set of reducible conditions of K_i*
Keep on applying construction principles to each reducible condition of K_i* until a DRS K_i is obtained that only contains irreducible conditions. Go to (i).

- (It's a *shift-reduce* algorithm, in case that means something to someone.)

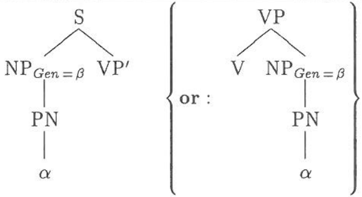
Segmentation

- $\boxed{\begin{array}{l} x_1, \ldots, x_n \\ \hline \phi_1 \\ \vdots \\ \phi_m \end{array}} \bigoplus S, S', \ldots \rightsquigarrow \boxed{\begin{array}{l} x_1, \ldots, x_n \\ \hline \phi_1 \\ \vdots \\ \phi_m \\ \mathrm{S} \end{array}} \bigoplus S', \ldots$
- If this stops before all S have been dealt with, the discourse is incoherent.

DRS Construction Algorithm: Names

CR.PN

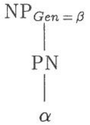
Triggering configuration $\gamma \subseteq \overline{\gamma} \in \mathbf{Con_K}$:



Introduce into the universe of the main DRS: new discourse referent **u**

Introduce into the condition set of the main DRS: new conditions $\alpha(\mathbf{u})$, $Gen(\mathbf{u}) = \beta$

Substitute in $\overline{\gamma}$: **u** for



Example: Names

Julian smiled. He saw a cat.

$$\rightsquigarrow \begin{array}{|c|}\hline \\ \hline \\ \\ \hline\end{array} \oplus \text{Julian smiled} \oplus \text{He saw a cat.}$$

$$\rightsquigarrow \begin{array}{|c|}\hline \\ \hline \text{Julian smiled.} \\ \hline\end{array} \oplus \text{He saw a cat.}$$

Example: Names

Julian smiled. He saw a cat.

$\rightsquigarrow \begin{array}{|c|}\hline \\ \hline \\ \hline\end{array}$ $\bigoplus$ Julian smiled $\bigoplus$ He saw a cat.

$\rightsquigarrow \begin{array}{|l|}\hline \\ \hline \text{Julian smiled.} \\ \hline\end{array}$ $\bigoplus$ He saw a cat.

$\rightsquigarrow \begin{array}{|l|}\hline j \\ \hline \text{Julian}(j) \\ \text{Gen}(j) = \text{m} \\ j \text{ smiled.} \\ \hline\end{array}$ $\bigoplus$ He saw a cat

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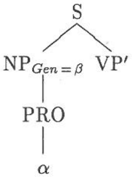
$\rightsquigarrow \begin{array}{|l|}\hline j \\ \hline \text{Julian}(j) \\ \text{Gen}(j) = \text{m} \\ j \text{ smiled.} \\ \hline \end{array} \oplus \text{He saw a cat} \rightsquigarrow \begin{array}{|l|}\hline j \\ \hline \text{Julian}(j) \\ \text{Gen}(j) = \text{m} \\ \texttt{smile}(j) \\ \hline \end{array} \oplus \text{He saw a cat}$

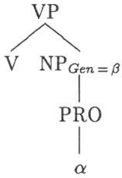
DRS Construction Algorithm: Pronouns

CR.PRO

Triggering configuration $\gamma \subseteq \overline{\gamma} \in \mathbf{Con_K}$**:**



or :

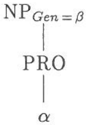


Choose suitable antecedent v, such that v is accessible and $Gen(\mathrm{v}) = \beta$

Introduce in $\mathbf{U_K}$**:** new discourse referent **u**

Introduce in $\mathbf{Con_K}$**:** new conditions $\mathbf{u} = \mathbf{v}$, $Gen(\mathbf{u}) = \beta$

Substitute in $\overline{\gamma}$**:** **u** for



Example: Pronouns

j
Julian(j) Gen(j) = m $\texttt{smile}$(j)

$\bigoplus$ He saw a cat.

Example: Pronouns

$$\begin{array}{|l|}\hline j \\ \hline \text{Julian}(j) \\ \text{Gen}(j) = \text{m} \\ \texttt{smile}(j) \\ \hline \end{array} \oplus \text{He saw a cat.}$$

$$\rightsquigarrow \begin{array}{|l|}\hline j \\ \hline \text{Julian}(j) \\ \text{Gen}(j) = \text{m} \\ \texttt{smile}(j) \\ \text{He saw a cat.} \\ \hline \end{array}$$

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$$\begin{array}{|l|}\hline j \\ \hline \text{Julian}(j) \\ \text{Gen}(j) = \text{m} \\ \texttt{smile}(j) \\ \hline \end{array} \oplus \text{He saw a cat.}$$

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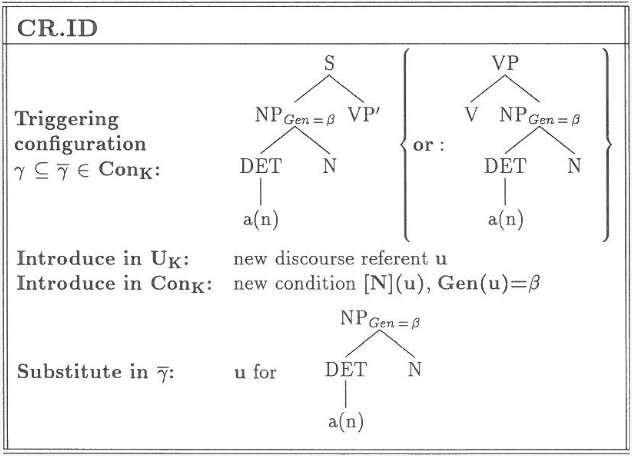
DRS Construction Algorithm: Indefinites



Example: Indefinites

j,u
Julian(j)
Gen(j) = m
`smile`(j)
u = j
Gen(u) = m
u saw a cat.

Example: Indefinites

j,u
Julian(*j*)
Gen(*j*) = m
`smile`(*j*)
u = j
Gen(*u*) = m
u saw a cat.

$\rightsquigarrow$

j,u,v
Julian(*j*)
Gen(*j*) = m
`smile`(*j*)
u = j
Gen(*u*) = m
Gen(*v*) = n
[cat](*v*)
u saw *v*.

Example: Indefinites

j,u
Julian(*j*)
Gen(*j*) = m
`smile`(*j*)
u = j
Gen(*u*) = m
u saw a cat.

$\leadsto$

j,u,v
Julian(*j*)
Gen(*j*) = m
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u = j
Gen(*u*) = m
Gen(*v*) = n
[cat](*v*)
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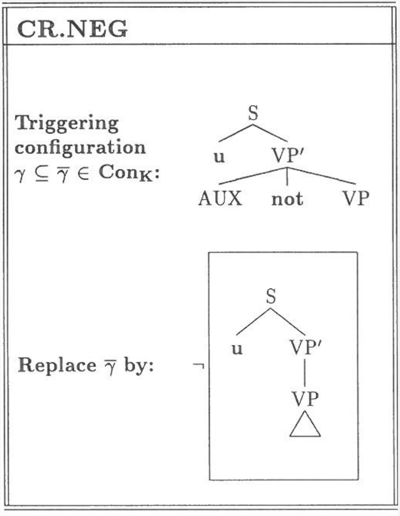
Gen(*v*) = n
`cat`(*v*)
u saw *v*.

DRS Construction Algorithm: Negation



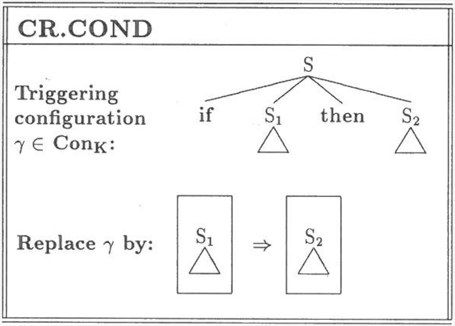
Example: Negation

(47) A man is not seeing a cat. He smiles, #it does not.

$$\begin{array}{|l|}\hline \\ \hline \text{A man is not seeing a cat} \\ \hline \end{array} \rightsquigarrow \begin{array}{|l|}\hline x \\ \hline \texttt{man}(x) \\ \neg \begin{array}{|l|}\hline \\ \hline x \text{ sees a cat} \\ \hline \end{array} \\ \hline \end{array}$$

$$\rightsquigarrow \begin{array}{|l|}\hline x \\ \hline \texttt{man}(x) \\ \neg \begin{array}{|l|}\hline y \\ \hline \texttt{cat}(y) \\ \texttt{see}(x, y) \\ \hline \end{array} \\ \hline \end{array}$$

DRS Construction Algorithm: Conditionals



Example: Conditionals

(48) If a farmer owns a donkey, he beats it.

$$
\begin{array}{|c|}
\hline
\\
\hline
\begin{array}{|c|}
\hline
\\
\hline
\text{A farmer owns a donkey} \\
\hline
\end{array}
\Rightarrow
\begin{array}{|c|}
\hline
\\
\hline
\text{He beats it} \\
\hline
\end{array} \\
\hline
\end{array}
\rightsquigarrow
$$

$$
\begin{array}{|c|}
\hline
\\
\hline
\begin{array}{|l|}
\hline
x, y \\
\hline
\texttt{farmer}(x) \\
\texttt{donkey}(y) \\
\texttt{owns}(x, y) \\
\hline
\end{array}
\Rightarrow
\begin{array}{|l|}
\hline
x', y' \\
\hline
\texttt{beat}(x', y') \\
x' = x \\
y' = y \\
\hline
\end{array} \\
\hline
\end{array}
$$

(49) Maria has three siblings, two of whom are female.
Her sisters are here. ✗He is somewhere else.

$$
\begin{array}{|l|}
\hline
m, sib, sis \\
\hline
\text{Maria}(m) \\
\texttt{siblings}(sib, m) \\
\#sib = 3 \\
\#sis = 2 \\
\texttt{part-of}(sis, sib) \\
\texttt{female}(sis) \\
\texttt{sister}(sis, m) \\
\texttt{here}(sis) \\
\neg \begin{array}{|l|}
\hline
\\
\hline
\texttt{here}(x) \\
Gen(x) = m \\
\hline
\end{array} \\
\hline
\end{array}
$$

But something is missing...

(50) Julian smiled. He saw a cat.

- This tells you something about why Julian is smiling.

But something is missing...

(50) Julian smiled. He saw a cat.

- This tells you something about why Julian is smiling.

(51) Julian smiled. He saw a horrible accident.

And what about these?

(52) Julian smiled because he saw a cat.

(53) Julian smiled, ??but he saw a cat.

(54) Julian smiled. ?Therefore he saw a cat.

And what about these?

(52) Julian smiled because he saw a cat.

(53) Julian smiled, ??but he saw a cat.

(54) Julian smiled. ?Therefore he saw a cat.

- There is much more to how sentences compose than is captured by DRT.