Logic in Action

Chapter 5: Logic, Information and Knowledge

http://www.logicinaction.org/

Observation, inference and communication

Someone is standing next to a room and sees a white object outside. Now another person tells her that there is an object inside the room of the same colour as the one outside. After all this, the first person reasons and get to know that there is a white object inside the room. This is based on three actions: an observation, then an act of communication, and finally an inference putting things together.

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From objective to subjective

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- If $p \to q$ and p are true, then q is true.

From objective to subjective

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- If $p \rightarrow q$ and p are true, then q is true.

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- If $p \to q$ and p are true, then q is true.

to

- If **I know** $p \to q$ and **I know** p, then **I know** q.

Representation

The key idea:

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The key idea:

Represent **uncertainty** rather than information.

Example (1)

Consider the uncertainty of an *agent*:

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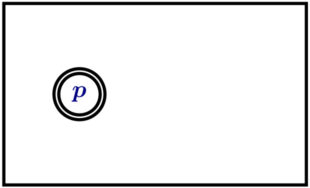

- ***p*** is the case

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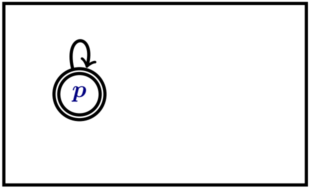

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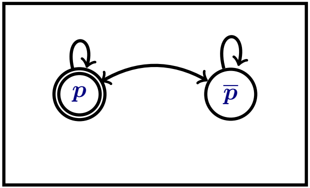

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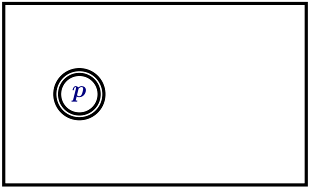

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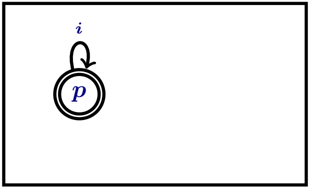

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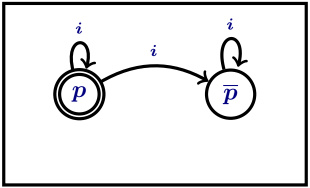

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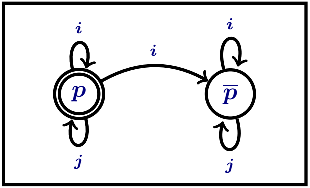

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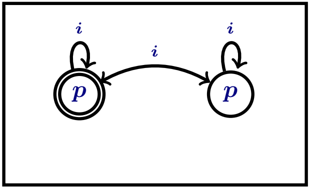

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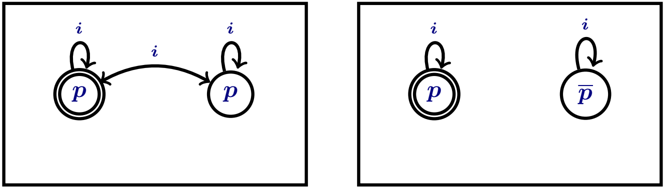

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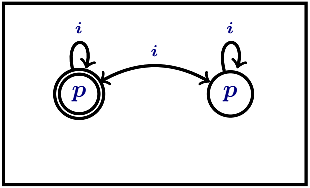

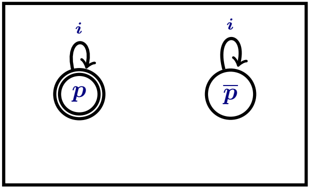

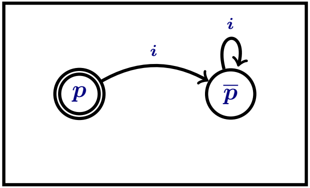

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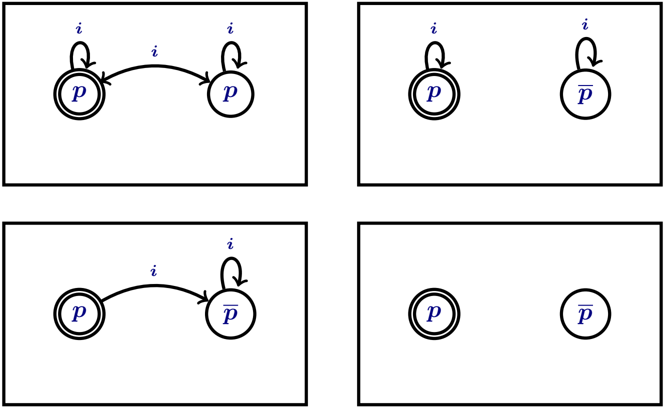

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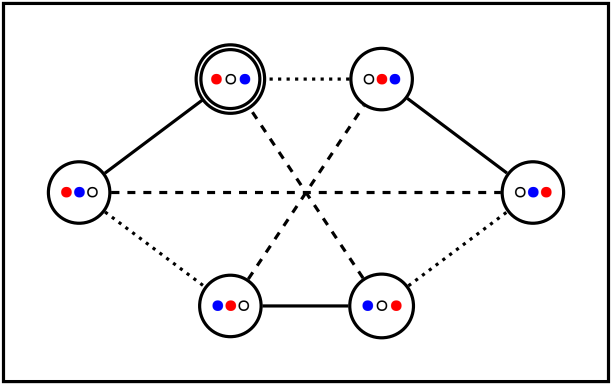

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The most basic of such changes:

Reduction of uncertainty means **more** information.

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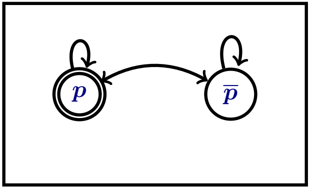

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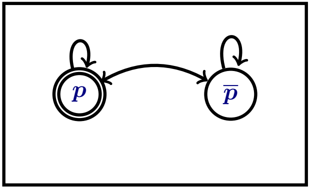

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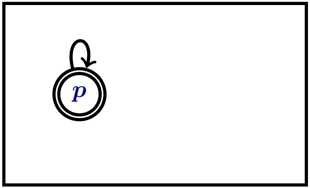

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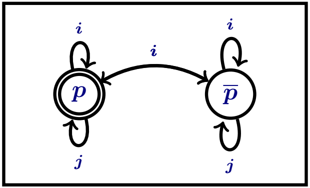

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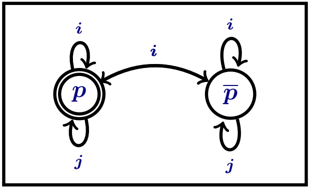

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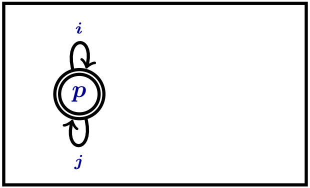

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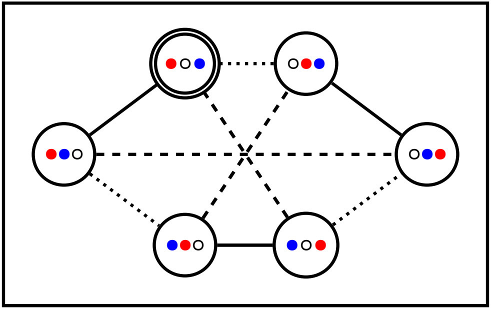

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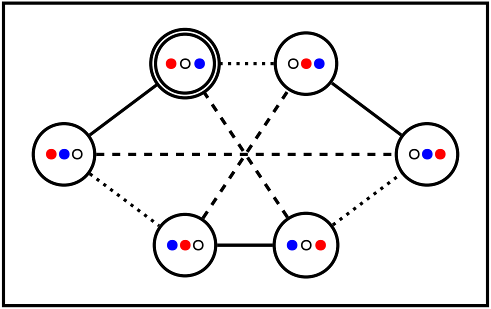

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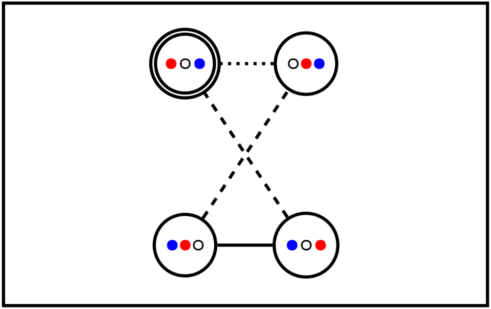

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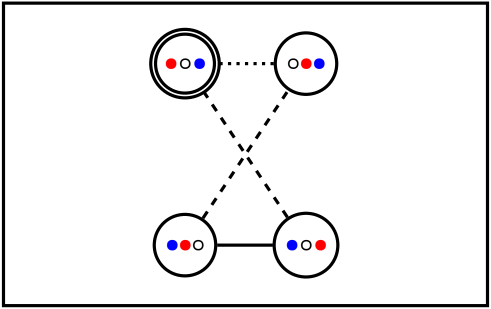

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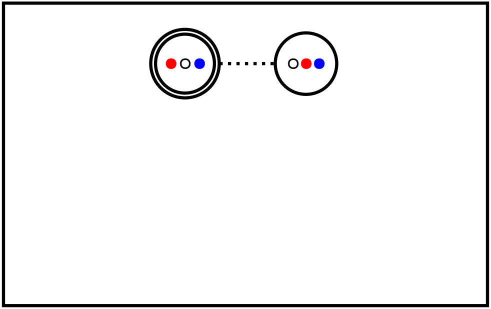

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We abbreviate $\neg\Box_i \neg\varphi$ as $\Diamond_i \varphi$.

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$$\Box_J\, (\Box_N\, r \;\lor\; \Box_N\, \neg r) \land (\neg\Box_J\, r \;\land\; \neg\Box_J\, \neg r)$$

To practice

1. James knows that it is raining.
2. Natalia knows whether it is raining.
3. James knows that Natalia knows whether it is raining, but he does not know it.
4. Natalia considers raining possible.
5. James does not know that it is raining, and actually it is not raining.
6. Natalia knows that it is raining, but in fact it is not raining.
7. James knows that if it is raining, the floor will be wet.
8. If James knows that if it is raining the floor will be wet, and he also knows that it is raining, then he knows that the floor is wet.
9. James considers possible that Natalia knows that it is raining.
10. Natalia does not know that James knows that she knows whether it is raining.

From natural to formal (1)

In this story we have three characters: Sherlock (S), Hemish (H) and James (J). Use the following notation:

- a - **“the doctor ate the fish”**
- d - **“the doctor died of poison”**
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1. Sherlock knows that the doctor died of poison.

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$$(\neg\Box_H\, d \land \neg\Box_H\, \neg d) \land \Diamond_H\, (\Box_S\, d \lor \Box_S\, \neg d)$$

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4. Hemish knows that if the fish was rotten and the doctor ate it (the fish), then he (the doctor) died of poison.

5. Sherlock knows that James knows whether he (James) put cyanide in the fish or not.

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$$\Box_J \Box_S d \wedge \Box_J \neg \Box_H d$$

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6. James knows Sherlock knows the doctor died of poison, and also knows that Hemish does not know it.

$$\Box_J \Box_S\, d \;\land\; \Box_J \neg \Box_H\, d$$

7. Sherlock knows that if James put cyanide in the fish, then he (James) knows it.

$$\Box_S \left(c \rightarrow \Box_J\, c\right)$$

From natural to formal (3)

From natural to formal (3)

8. Sherlock knows that Hemish does not know that the fish was rotten.

9. James knows that the fish was rotten and that he put cyanide in the fish.

10. No one knows the doctor did not eat the fish.

From natural to formal (3)

8. Sherlock knows that Hemish does not know that the fish was rotten.

$$\Box_S \neg \Box_H r$$

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$$\Box_J (r \wedge c)$$

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10. No one knows the doctor did not eat the fish.

$$\neg \Box_S \neg a \;\land\; \neg \Box_H \neg a \;\land\; \neg \Box_J \neg a$$

From formal to natural (1)

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$\Box_S \neg \Box_J a$

$\Box_H \Big((a \wedge (c \vee r)) \rightarrow d \Big)$

$\Box_J (c \wedge \neg \Box_S c \wedge \neg \Box_S \neg c)$

$\neg(\Box_S r \wedge \Box_H r \wedge \Box_J r)$

$\Box_J \Diamond_H (r \wedge a)$

From formal to natural (1)

$\Box_S \neg \Box_J a$

Sherlock knows that James does not know the doctor ate the fish.

$\Box_H \big((a \wedge (c \vee r)) \to d \big)$

$\Box_J \, (c \wedge \neg \Box_S c \wedge \neg \Box_S \neg c)$

$\neg (\Box_S r \wedge \Box_H r \wedge \Box_J r)$

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From formal to natural (1)

$\Box_S \neg \Box_J a$	Sherlock knows that James does not know the doctor ate the fish.
$\Box_H \big((a \wedge (c \vee r)) \rightarrow d \big)$	Hemish knows that if the doctor ate the fish and this was rotted or with cyanide, then the doctor died of poison.
$\Box_J (c \wedge \neg \Box_S c \wedge \neg \Box_S \neg c)$	
$\neg (\Box_S r \wedge \Box_H r \wedge \Box_J r)$	
$\Box_J \Diamond_H (r \wedge a)$	

From formal to natural (1)

$\Box_S \neg \Box_J a$	Sherlock knows that James does not know the doctor ate the fish.
$\Box_H \big((a \land (c \lor r)) \to d\big)$	Hemish knows that if the doctor ate the fish and this was rotted or with cyanide, then the doctor died of poison.
$\Box_J (c \land \neg \Box_S c \land \neg \Box_S \neg c)$	James knows that he put cyanide in the fish, and that Sherlock does not know whether this happened or not.
$\neg(\Box_S r \land \Box_H r \land \Box_J r)$	
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$\neg (\Box_S r \land \Box_H r \land \Box_J r)$	Not everyone knows the fish was rotten.
$\Box_J \Diamond_H (r \land a)$	James knows Hemish considers possible the doctor ate the rotten fish.

From formal to natural (2)

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$$\neg\Box_S \Box_H c \;\wedge\; \Diamond_S \Box_H c$$

$$d \longrightarrow (\Diamond_S c \wedge \Diamond_H c)$$

$$\Box_J \Big(d \longrightarrow (\Diamond_S c \;\wedge\; \neg\Diamond_S r)\Big)$$

From formal to natural (2)

$$\neg\Box_S \Box_H c \;\land\; \Diamond_S \Box_H c$$

Sherlock does not know that Hemish knows that James put cyanide in the fish, but he (Sherlock) considers possible that James knows it.

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Sherlock does not know that Hemish knows that James put cyanide in the fish, but he (Sherlock) considers possible that James knows it.

$d \rightarrow (\Diamond_S c \wedge \Diamond_H c)$

If the doctor died of poison, then Sherlock and Hemish consider possible that James put cyanide in the fish.

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$\Box_J \Big(d \rightarrow (\Diamond_S c \; \wedge \; \neg \Diamond_S r) \Big)$

James knows that if the doctor died of poison, then Sherlock considers possible that he (James) put cyanide in the fish, but not that the fish was rotten.

From formal to natural (3)

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$$\Box_J\,(r \rightarrow (d \wedge \Box_H\, d)) \wedge \neg\Diamond_S\, \neg c$$

$$\Box_H\,(\Box_S\, d \rightarrow d) \wedge \Box_H\,(\Box_H\, d \rightarrow \Diamond_S\, \neg d)$$

From formal to natural (3)

$$\Box_J \left(r \rightarrow (d \land \Box_H d)\right) \land \neg\Diamond_S \neg c$$

James knows that if the fish was rotten, then doctor died of poison and Hemish knows it (that the doctor died of poison) but Sherlock does not consider possible that James did not put cyanide in the fish.

$$\Box_H \left(\Box_S d \rightarrow d\right) \land \Box_H \left(\Box_H d \rightarrow \Diamond_S \neg d\right)$$

From formal to natural (3)

$$\Box_J\,(r \rightarrow (d \wedge \Box_H\, d)) \wedge \neg\Diamond_S\,\neg c$$

James knows that if the fish was rotten, then doctor died of poison and Hemish knows it (that the doctor died of poison) but Sherlock does not consider possible that James did not put cyanide in the fish.

$$\Box_H\,(\Box_S\, d \rightarrow d) \wedge \Box_H\,(\Box_H\, d \rightarrow \Diamond_S\,\neg d)$$

Hemish knows that if Sherlock knows the doctor died of poison, then the doctor indeed died of poison, but he (Hemish) also knows that if he (Hemish) knows the doctor died of poison, then Sherlock considers possible that the doctor did not died of poison.

The models

The structures in which we evaluate modal formulas, **relational structures**, have three components:

$$\boldsymbol{M} = \langle \quad , \quad , \quad \rangle$$

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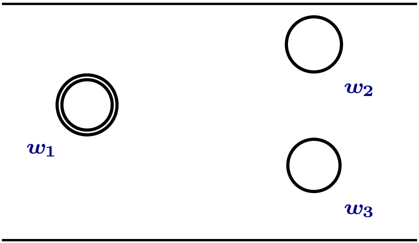

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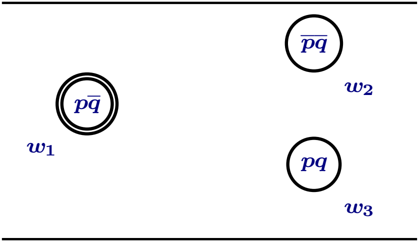

$$M = \langle W, \quad , V \rangle$$

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- a non-empty set W of **situations** or **worlds** (with a distinguished one),
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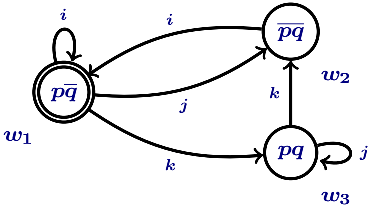
- an **accessibility** relation R_i for each agent i.



$$M = \langle W, R_i, V \rangle$$

Properties of the relation

Each **accessibility** relation $\boldsymbol{R}$ may have some special properties.

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- **Equivalence.** If it is reflexive, transitive and symmetric.
- **Euclidity.** For all worlds w, v and u, if Rwv and Rwu then Rvu.

Deciding truth-value of formulas

Take a relational structure $\boldsymbol{M} = \langle \boldsymbol{W}, \boldsymbol{R_i}, \boldsymbol{V} \rangle$, and pick a world $\boldsymbol{w} \in \boldsymbol{W}$:

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$$
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&\text{iff} \quad \text{not} \left((M, w) \models \Box_i \neg\varphi \right)
\end{aligned}
$$

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& \text{iff} \quad \text{not} \left((M, w) \models \Box_i \neg\varphi \right) \\
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$(M, w) \models \Diamond_i \varphi$ iff $(M, w) \models \neg\Box_i \neg\varphi$

iff not $\left((M, w) \models \Box_i \neg\varphi \right)$

iff not $\left(\text{for all } u \in W, \text{ if } R_i wu \text{ then } (M, u) \models \neg\varphi \right)$

iff there is a $u \in W$ such that $R_i wu$ and not $(M, u) \models \neg\varphi$

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\begin{array}{lll}
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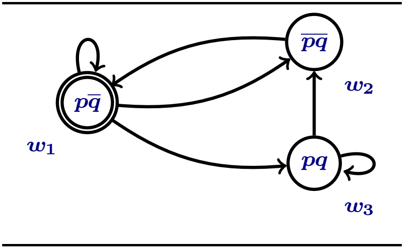
$$

To practice (1)



M

$(M, w_1) \models \Diamond \neg p$?	$(M, w_2) \models \Diamond \neg p$?	$(M, w_3) \models \Diamond \neg p$?
$(M, w_1) \models \Box (p \leftrightarrow q)$?	$(M, w_2) \models \Box (p \leftrightarrow q)$?	$(M, w_3) \models \Box (p \leftrightarrow q)$?
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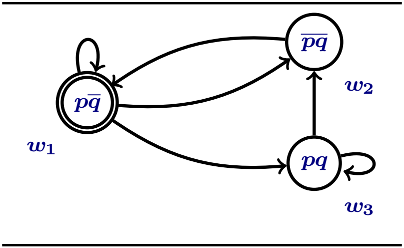

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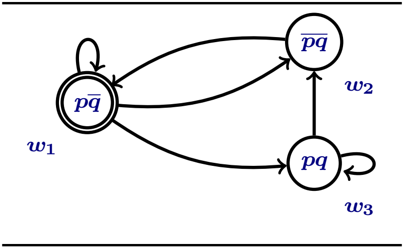

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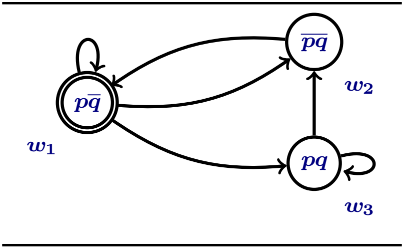

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$(M, w_1) \models \Box (p \leftrightarrow q)$	✗	$(M, w_2) \models \Box (p \leftrightarrow q)$	?	$(M, w_3) \models \Box (p \leftrightarrow q)$	?
$(M, w_1) \models p \vee \Box p$	✓	$(M, w_2) \models p \vee \Box p$	?	$(M, w_3) \models p \vee \Box p$	?

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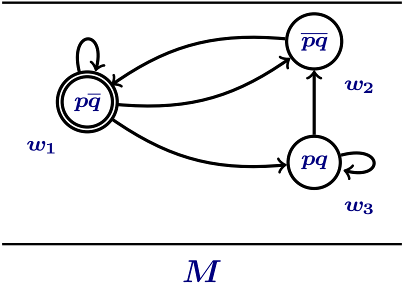

$(M, w_1) \models \Diamond \neg p$	✓	$(M, w_2) \models \Diamond \neg p$	✗	$(M, w_3) \models \Diamond \neg p$	?
$(M, w_1) \models \Box (p \leftrightarrow q)$	✗	$(M, w_2) \models \Box (p \leftrightarrow q)$	?	$(M, w_3) \models \Box (p \leftrightarrow q)$	?
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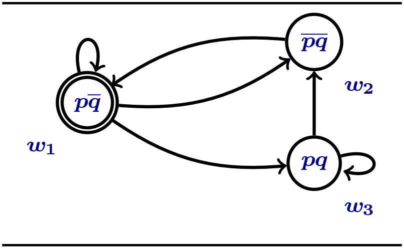
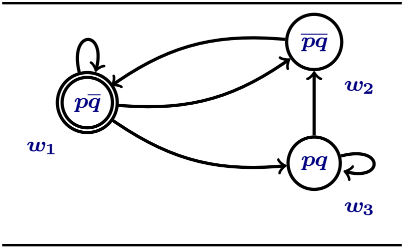


M

$(M, w_1) \models \Diamond \neg p$ ✓	$(M, w_2) \models \Diamond \neg p$ ✗	$(M, w_3) \models \Diamond \neg p$?
$(M, w_1) \models \Box (p \leftrightarrow q)$ ✗	$(M, w_2) \models \Box (p \leftrightarrow q)$ ✗	$(M, w_3) \models \Box (p \leftrightarrow q)$?
$(M, w_1) \models p \vee \Box p$ ✓	$(M, w_2) \models p \vee \Box p$?	$(M, w_3) \models p \vee \Box p$?

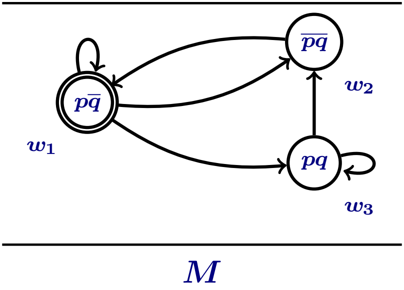
To practice (1)



M

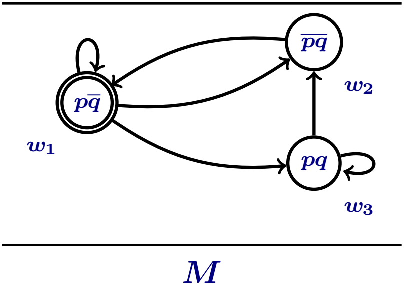
$(M, w_1) \models \Diamond \neg p$	✓	$(M, w_2) \models \Diamond \neg p$	✗	$(M, w_3) \models \Diamond \neg p$	?
$(M, w_1) \models \Box (p \leftrightarrow q)$	✗	$(M, w_2) \models \Box (p \leftrightarrow q)$	✗	$(M, w_3) \models \Box (p \leftrightarrow q)$	?
$(M, w_1) \models p \vee \Box p$	✓	$(M, w_2) \models p \vee \Box p$	✓	$(M, w_3) \models p \vee \Box p$	?

To practice (1)



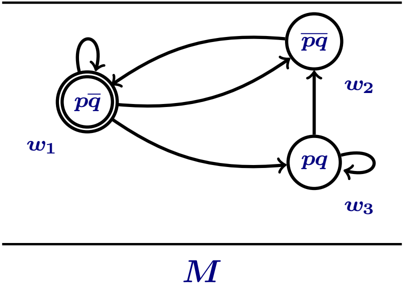
$(M, w_1) \models \Diamond \neg p$ ✓	$(M, w_2) \models \Diamond \neg p$ ✗	$(M, w_3) \models \Diamond \neg p$ ✓
$(M, w_1) \models \Box (p \leftrightarrow q)$ ✗	$(M, w_2) \models \Box (p \leftrightarrow q)$ ✗	$(M, w_3) \models \Box (p \leftrightarrow q)$?
$(M, w_1) \models p \vee \Box p$ ✓	$(M, w_2) \models p \vee \Box p$ ✓	$(M, w_3) \models p \vee \Box p$?

To practice (1)



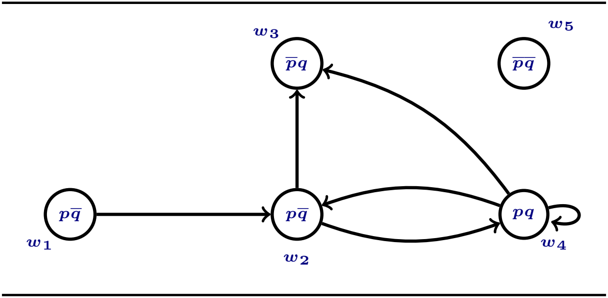
$(M, w_1) \models \Diamond \neg p$	✓	$(M, w_2) \models \Diamond \neg p$	✗	$(M, w_3) \models \Diamond \neg p$	✓
$(M, w_1) \models \Box (p \leftrightarrow q)$	✗	$(M, w_2) \models \Box (p \leftrightarrow q)$	✗	$(M, w_3) \models \Box (p \leftrightarrow q)$	✓
$(M, w_1) \models p \vee \Box p$	✓	$(M, w_2) \models p \vee \Box p$	✓	$(M, w_3) \models p \vee \Box p$	?

To practice (1)



$(M, w_1) \models \Diamond \neg p$ ✓	$(M, w_2) \models \Diamond \neg p$ ✗	$(M, w_3) \models \Diamond \neg p$ ✓
$(M, w_1) \models \Box (p \leftrightarrow q)$ ✗	$(M, w_2) \models \Box (p \leftrightarrow q)$ ✗	$(M, w_3) \models \Box (p \leftrightarrow q)$ ✓
$(M, w_1) \models p \vee \Box p$ ✓	$(M, w_2) \models p \vee \Box p$ ✓	$(M, w_3) \models p \vee \Box p$ ✓

To practice (2)

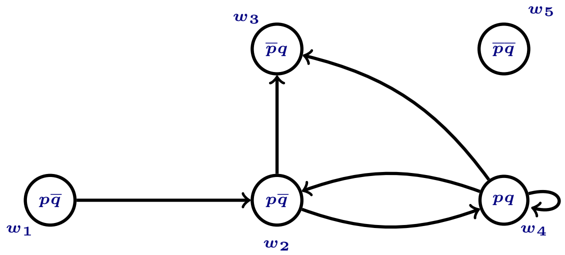


M

Indicate the worlds in which the following formulas are true.

$\Diamond q$	$\Box p$
$\Box p \rightarrow p$	$\Diamond \Diamond p \rightarrow \Diamond p$
$q \rightarrow \Box \Diamond q$	$\Diamond \Box p \rightarrow \Box \Diamond p$
$\Diamond (p \rightarrow q)$	$\Diamond (\neg p \wedge \neg q)$

To practice (2)

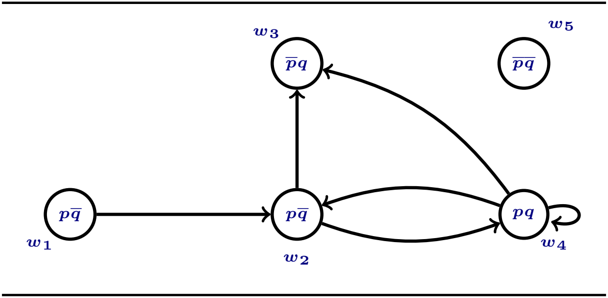


M

Indicate the worlds in which the following formulas are true.

$\Diamond q$	$\{w_2, w_4\}$	$\Box p$
$\Box p \rightarrow p$		$\Diamond\Diamond p \rightarrow \Diamond p$
$q \rightarrow \Box\Diamond q$		$\Diamond\Box p \rightarrow \Box\Diamond p$
$\Diamond(p \rightarrow q)$		$\Diamond(\neg p \wedge \neg q)$

To practice (2)

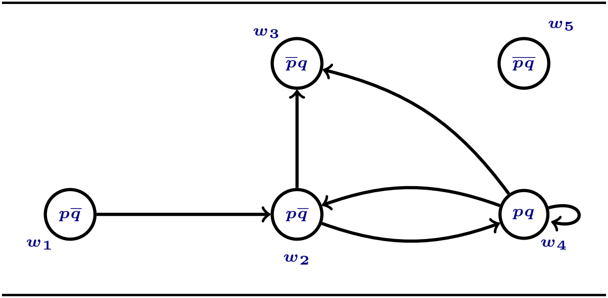


M

Indicate the worlds in which the following formulas are true.

$\Diamond q$	$\{w_2, w_4\}$	$\Box p$	$\{w_1, w_3, w_5\}$
$\Box p \rightarrow p$		$\Diamond \Diamond p \rightarrow \Diamond p$	
$q \rightarrow \Box \Diamond q$		$\Diamond \Box p \rightarrow \Box \Diamond p$	
$\Diamond (p \rightarrow q)$		$\Diamond (\neg p \wedge \neg q)$	

To practice (2)

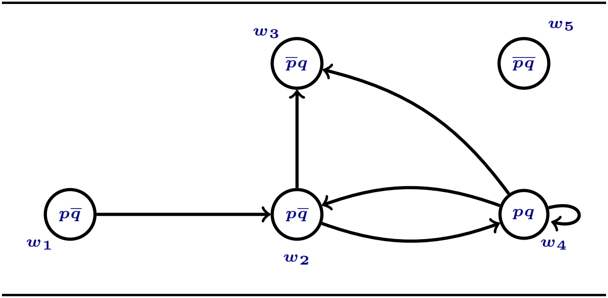


M

Indicate the worlds in which the following formulas are true.

$\Diamond q$	$\{w_2, w_4\}$	$\Box p$	$\{w_1, w_3, w_5\}$
$\Box p \rightarrow p$	$\{w_1, w_2, w_4\}$	$\Diamond\Diamond p \rightarrow \Diamond p$	
$q \rightarrow \Box\Diamond q$		$\Diamond\Box p \rightarrow \Box\Diamond p$	
$\Diamond(p \rightarrow q)$		$\Diamond(\neg p \wedge \neg q)$	

To practice (2)

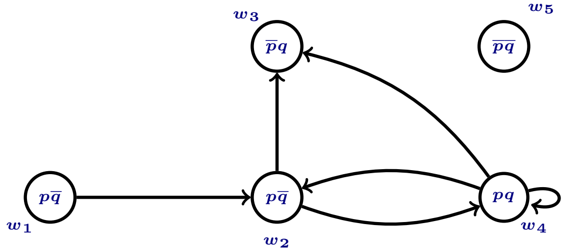


M

Indicate the worlds in which the following formulas are true.

$\Diamond q$	$\{w_2, w_4\}$	$\Box p$	$\{w_1, w_3, w_5\}$
$\Box p \rightarrow p$	$\{w_1, w_2, w_4\}$	$\Diamond \Diamond p \rightarrow \Diamond p$	$\{w_1, w_2, w_3, w_4, w_5\}$
$q \rightarrow \Box \Diamond q$		$\Diamond \Box p \rightarrow \Box \Diamond p$	
$\Diamond (p \rightarrow q)$		$\Diamond (\neg p \wedge \neg q)$	

To practice (2)

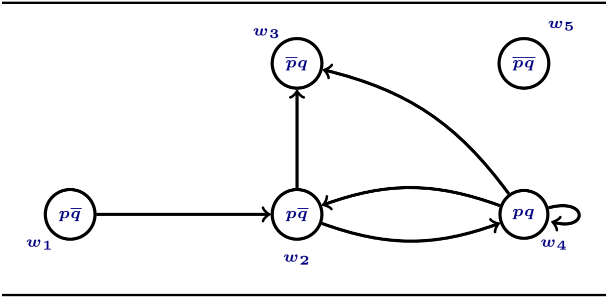


M

Indicate the worlds in which the following formulas are true.

$\Diamond q$	$\{w_2, w_4\}$	$\Box p$	$\{w_1, w_3, w_5\}$
$\Box p \rightarrow p$	$\{w_1, w_2, w_4\}$	$\Diamond \Diamond p \rightarrow \Diamond p$	$\{w_1, w_2, w_3, w_4, w_5\}$
$q \rightarrow \Box \Diamond q$	$\{w_1, w_2, w_3, w_5\}$	$\Diamond \Box p \rightarrow \Box \Diamond p$	
$\Diamond (p \rightarrow q)$		$\Diamond (\neg p \wedge \neg q)$	

To practice (2)

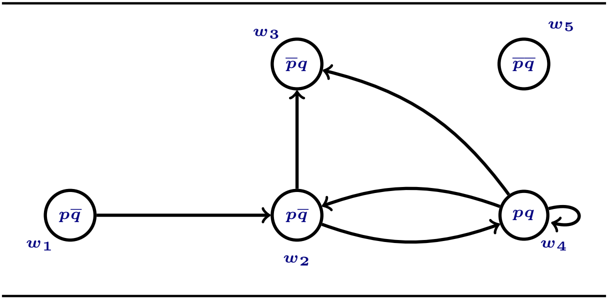


M

Indicate the worlds in which the following formulas are true.

$\Diamond q$	$\{w_2, w_4\}$	$\Box p$	$\{w_1, w_3, w_5\}$
$\Box p \rightarrow p$	$\{w_1, w_2, w_4\}$	$\Diamond \Diamond p \rightarrow \Diamond p$	$\{w_1, w_2, w_3, w_4, w_5\}$
$q \rightarrow \Box \Diamond q$	$\{w_1, w_2, w_3, w_5\}$	$\Diamond \Box p \rightarrow \Box \Diamond p$	$\{w_1, w_3, w_5\}$
$\Diamond (p \rightarrow q)$		$\Diamond (\neg p \wedge \neg q)$	

To practice (2)

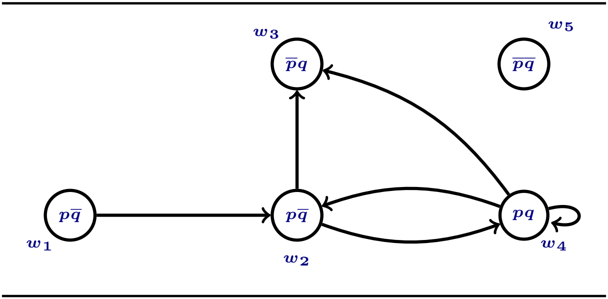


M

Indicate the worlds in which the following formulas are true.

$\Diamond q$	$\{w_2, w_4\}$	$\Box p$	$\{w_1, w_3, w_5\}$
$\Box p \rightarrow p$	$\{w_1, w_2, w_4\}$	$\Diamond \Diamond p \rightarrow \Diamond p$	$\{w_1, w_2, w_3, w_4, w_5\}$
$q \rightarrow \Box \Diamond q$	$\{w_1, w_2, w_3, w_5\}$	$\Diamond \Box p \rightarrow \Box \Diamond p$	$\{w_1, w_3, w_5\}$
$\Diamond (p \rightarrow q)$	$\{w_2, w_4\}$	$\Diamond (\neg p \wedge \neg q)$	

To practice (2)

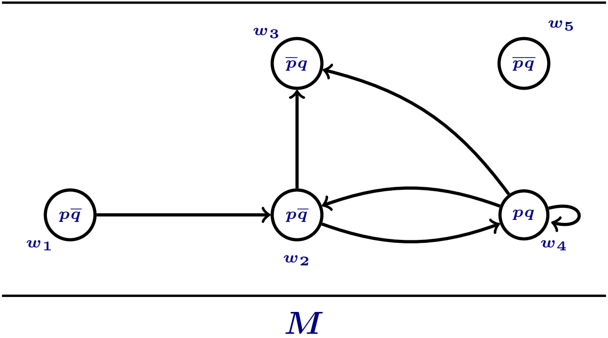


M

Indicate the worlds in which the following formulas are true.

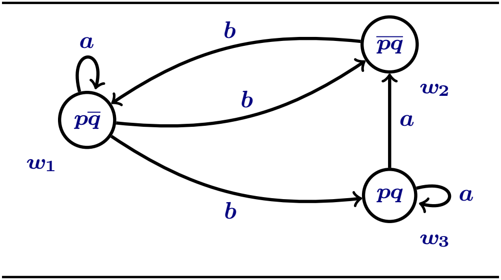
$\Diamond q$	$\{w_2, w_4\}$	$\Box p$	$\{w_1, w_3, w_5\}$
$\Box p \rightarrow p$	$\{w_1, w_2, w_4\}$	$\Diamond \Diamond p \rightarrow \Diamond p$	$\{w_1, w_2, w_3, w_4, w_5\}$
$q \rightarrow \Box \Diamond q$	$\{w_1, w_2, w_3, w_5\}$	$\Diamond \Box p \rightarrow \Box \Diamond p$	$\{w_1, w_3, w_5\}$
$\Diamond (p \rightarrow q)$	$\{w_2, w_4\}$	$\Diamond (\neg p \wedge \neg q)$	$\{\ \}$

To practice (3)



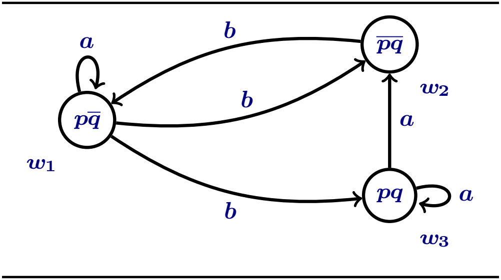
For each world in the model, provide a formula that is true only in that world and false in all the others.

Multiple relations



M

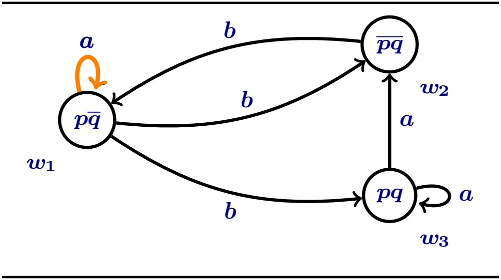
Multiple relations



M

$(M, w_1) \models \Diamond_a \neg p$?	$(M, w_2) \models \Diamond_a \neg p$?	$(M, w_3) \models \Diamond_a \neg p$?
$(M, w_1) \models \Box_b (p \leftrightarrow q)$?	$(M, w_2) \models \Box_b (p \leftrightarrow q)$?	$(M, w_3) \models \Box_b (p \leftrightarrow q)$?
$(M, w_1) \models \Box_b p \vee \Diamond_a q$?	$(M, w_2) \models \Box_b p \vee \Diamond_a q$?	$(M, w_3) \models \Box_b p \vee \Diamond_a q$?

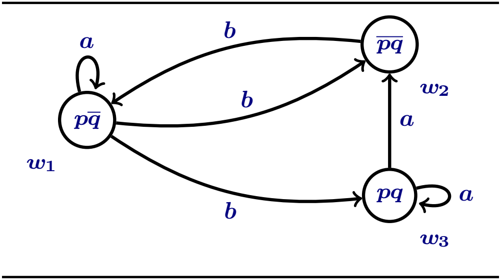
Multiple relations



M

$(M, w_1) \models \Diamond_a \neg p$?	$(M, w_2) \models \Diamond_a \neg p$?	$(M, w_3) \models \Diamond_a \neg p$?
$(M, w_1) \models \Box_b (p \leftrightarrow q)$?	$(M, w_2) \models \Box_b (p \leftrightarrow q)$?	$(M, w_3) \models \Box_b (p \leftrightarrow q)$?
$(M, w_1) \models \Box_b p \lor \Diamond_a q$?	$(M, w_2) \models \Box_b p \lor \Diamond_a q$?	$(M, w_3) \models \Box_b p \lor \Diamond_a q$?

Multiple relations



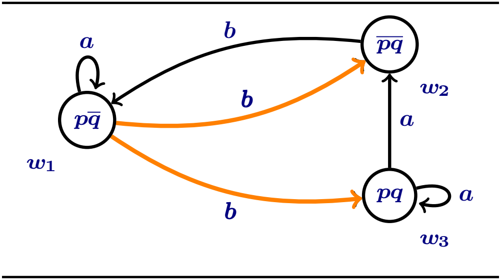
M

$(M, w_1) \models \Diamond_a \neg p$ ✗ $(M, w_2) \models \Diamond_a \neg p$? $(M, w_3) \models \Diamond_a \neg p$?

$(M, w_1) \models \Box_b (p \leftrightarrow q)$? $(M, w_2) \models \Box_b (p \leftrightarrow q)$? $(M, w_3) \models \Box_b (p \leftrightarrow q)$?

$(M, w_1) \models \Box_b p \vee \Diamond_a q$? $(M, w_2) \models \Box_b p \vee \Diamond_a q$? $(M, w_3) \models \Box_b p \vee \Diamond_a q$?

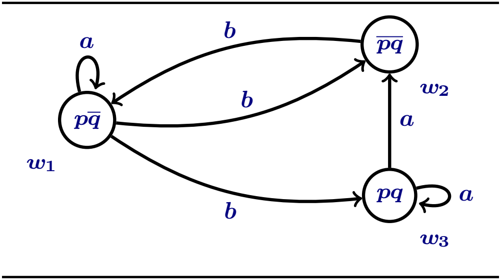
Multiple relations



M

$(M, w_1) \models \Diamond_a \neg p$ ✗	$(M, w_2) \models \Diamond_a \neg p$?	$(M, w_3) \models \Diamond_a \neg p$?
$(M, w_1) \models \Box_b (p \leftrightarrow q)$?	$(M, w_2) \models \Box_b (p \leftrightarrow q)$?	$(M, w_3) \models \Box_b (p \leftrightarrow q)$?
$(M, w_1) \models \Box_b p \vee \Diamond_a q$?	$(M, w_2) \models \Box_b p \vee \Diamond_a q$?	$(M, w_3) \models \Box_b p \vee \Diamond_a q$?

Multiple relations



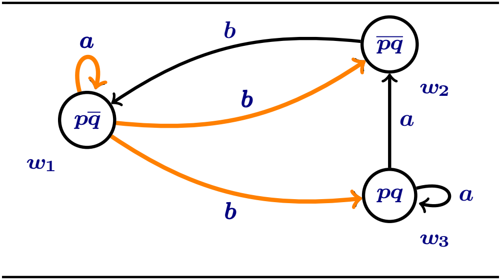
M

$(M, w_1) \models \Diamond_a \neg p$ ✗ $(M, w_2) \models \Diamond_a \neg p$? $(M, w_3) \models \Diamond_a \neg p$?

$(M, w_1) \models \Box_b (p \leftrightarrow q)$ ✓ $(M, w_2) \models \Box_b (p \leftrightarrow q)$? $(M, w_3) \models \Box_b (p \leftrightarrow q)$?

$(M, w_1) \models \Box_b p \vee \Diamond_a q$? $(M, w_2) \models \Box_b p \vee \Diamond_a q$? $(M, w_3) \models \Box_b p \vee \Diamond_a q$?

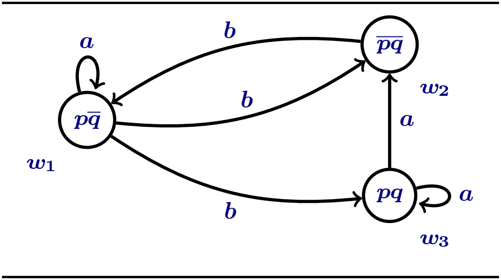
Multiple relations



M

$(M, w_1) \models \Diamond_a \neg p$ ✗	$(M, w_2) \models \Diamond_a \neg p$?	$(M, w_3) \models \Diamond_a \neg p$?
$(M, w_1) \models \Box_b (p \leftrightarrow q)$ ✓	$(M, w_2) \models \Box_b (p \leftrightarrow q)$?	$(M, w_3) \models \Box_b (p \leftrightarrow q)$?
$(M, w_1) \models \Box_b p \lor \Diamond_a q$?	$(M, w_2) \models \Box_b p \lor \Diamond_a q$?	$(M, w_3) \models \Box_b p \lor \Diamond_a q$?

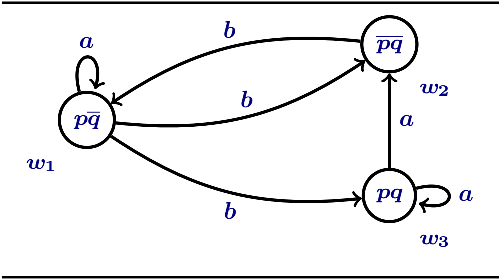
Multiple relations



M

$(M, w_1) \models \Diamond_a \neg p$ ✗	$(M, w_2) \models \Diamond_a \neg p$?	$(M, w_3) \models \Diamond_a \neg p$?
$(M, w_1) \models \Box_b (p \leftrightarrow q)$ ✓	$(M, w_2) \models \Box_b (p \leftrightarrow q)$?	$(M, w_3) \models \Box_b (p \leftrightarrow q)$?
$(M, w_1) \models \Box_b p \vee \Diamond_a q$ ✗	$(M, w_2) \models \Box_b p \vee \Diamond_a q$?	$(M, w_3) \models \Box_b p \vee \Diamond_a q$?

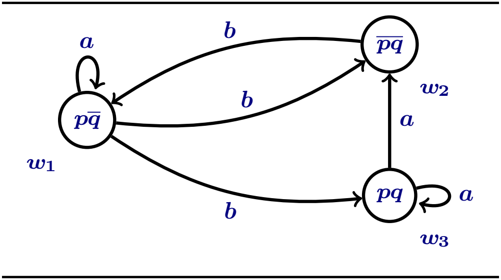
Multiple relations



M

$(M, w_1) \models \Diamond_a \neg p$ ✗	$(M, w_2) \models \Diamond_a \neg p$?	$(M, w_3) \models \Diamond_a \neg p$?
$(M, w_1) \models \Box_b (p \leftrightarrow q)$ ✓	$(M, w_2) \models \Box_b (p \leftrightarrow q)$?	$(M, w_3) \models \Box_b (p \leftrightarrow q)$?
$(M, w_1) \models \Box_b p \vee \Diamond_a q$ ✗	$(M, w_2) \models \Box_b p \vee \Diamond_a q$?	$(M, w_3) \models \Box_b p \vee \Diamond_a q$?

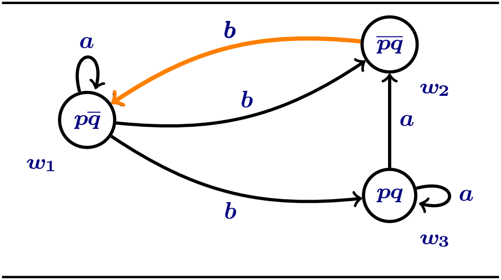
Multiple relations



M

$(M, w_1) \models \Diamond_a \neg p$ ✗	$(M, w_2) \models \Diamond_a \neg p$ ✗	$(M, w_3) \models \Diamond_a \neg p$?
$(M, w_1) \models \Box_b (p \leftrightarrow q)$ ✓	$(M, w_2) \models \Box_b (p \leftrightarrow q)$?	$(M, w_3) \models \Box_b (p \leftrightarrow q)$?
$(M, w_1) \models \Box_b p \vee \Diamond_a q$ ✗	$(M, w_2) \models \Box_b p \vee \Diamond_a q$?	$(M, w_3) \models \Box_b p \vee \Diamond_a q$?

Multiple relations



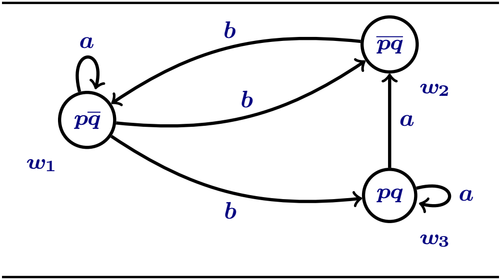
M

$(M, w_1) \models \Diamond_a \neg p$ ✗ $(M, w_2) \models \Diamond_a \neg p$ ✗ $(M, w_3) \models \Diamond_a \neg p$?

$(M, w_1) \models \Box_b (p \leftrightarrow q)$ ✓ $(M, w_2) \models \Box_b (p \leftrightarrow q)$? $(M, w_3) \models \Box_b (p \leftrightarrow q)$?

$(M, w_1) \models \Box_b p \vee \Diamond_a q$ ✗ $(M, w_2) \models \Box_b p \vee \Diamond_a q$? $(M, w_3) \models \Box_b p \vee \Diamond_a q$?

Multiple relations



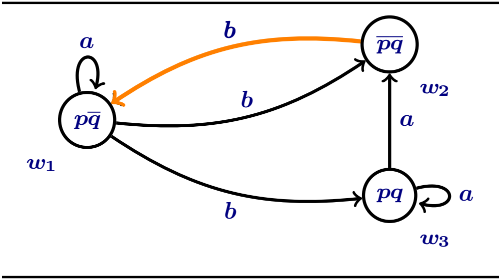
M

$(M, w_1) \models \Diamond_a \neg p$ ✗ $(M, w_2) \models \Diamond_a \neg p$ ✗ $(M, w_3) \models \Diamond_a \neg p$?

$(M, w_1) \models \Box_b (p \leftrightarrow q)$ ✓ $(M, w_2) \models \Box_b (p \leftrightarrow q)$ ✗ $(M, w_3) \models \Box_b (p \leftrightarrow q)$?

$(M, w_1) \models \Box_b\, p \vee \Diamond_a\, q$ ✗ $(M, w_2) \models \Box_b\, p \vee \Diamond_a\, q$? $(M, w_3) \models \Box_b\, p \vee \Diamond_a\, q$?

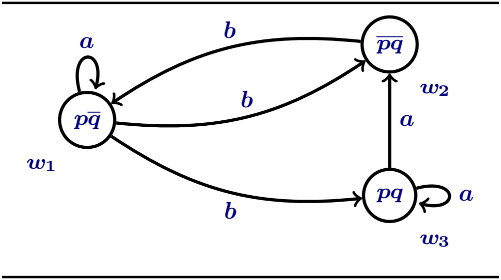
Multiple relations



M

$(M, w_1) \models \Diamond_a \neg p$ ✗	$(M, w_2) \models \Diamond_a \neg p$ ✗	$(M, w_3) \models \Diamond_a \neg p$?
$(M, w_1) \models \Box_b (p \leftrightarrow q)$ ✓	$(M, w_2) \models \Box_b (p \leftrightarrow q)$ ✗	$(M, w_3) \models \Box_b (p \leftrightarrow q)$?
$(M, w_1) \models \Box_b\, p \vee \Diamond_a\, q$ ✗	$(M, w_2) \models \Box_b\, p \vee \Diamond_a\, q$?	$(M, w_3) \models \Box_b\, p \vee \Diamond_a\, q$?

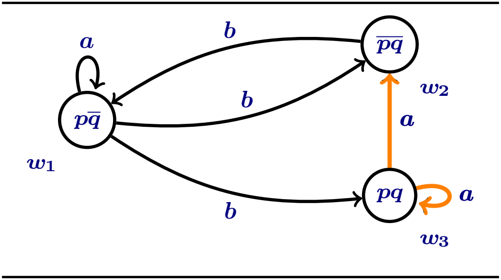
Multiple relations



M

$(M, w_1) \models \Diamond_a \neg p$ ✗	$(M, w_2) \models \Diamond_a \neg p$ ✗	$(M, w_3) \models \Diamond_a \neg p$?
$(M, w_1) \models \Box_b (p \leftrightarrow q)$ ✓	$(M, w_2) \models \Box_b (p \leftrightarrow q)$ ✗	$(M, w_3) \models \Box_b (p \leftrightarrow q)$?
$(M, w_1) \models \Box_b p \vee \Diamond_a q$ ✗	$(M, w_2) \models \Box_b p \vee \Diamond_a q$ ✓	$(M, w_3) \models \Box_b p \vee \Diamond_a q$?

Multiple relations



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$(M, w_1) \models \Box_b (p \leftrightarrow q)$	✓	$(M, w_2) \models \Box_b (p \leftrightarrow q)$	✗	$(M, w_3) \models \Box_b (p \leftrightarrow q)$	?
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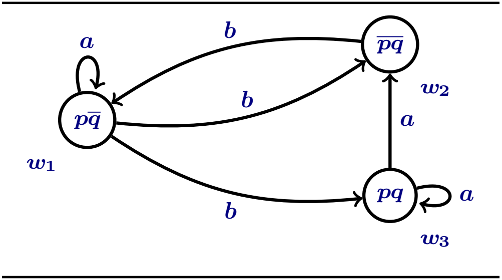

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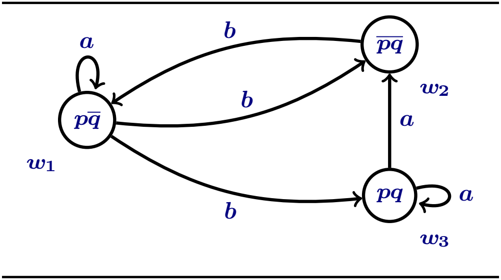

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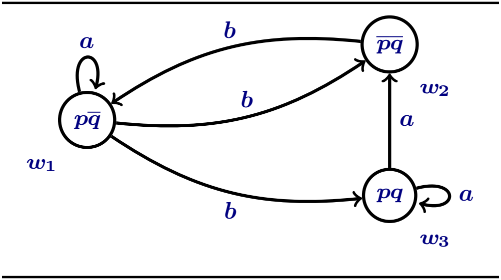

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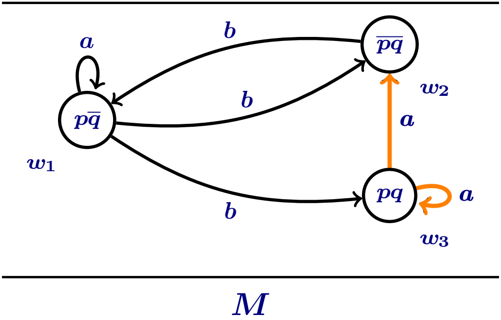

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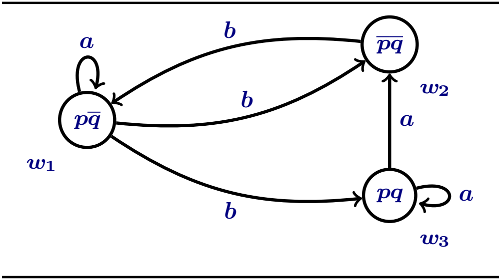

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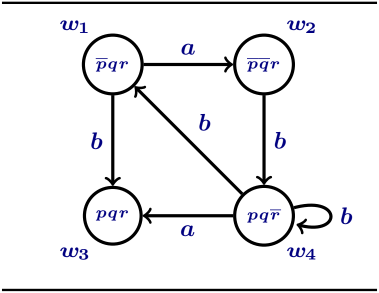
To practice



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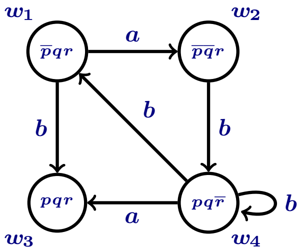

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Indicate the worlds in which the following formulas are true.

$\Diamond_a \Diamond_b p$	$\Box_a \Box_b r$
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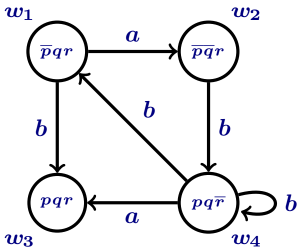

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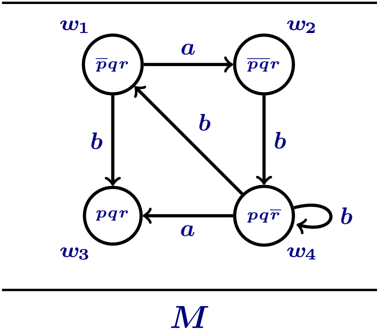

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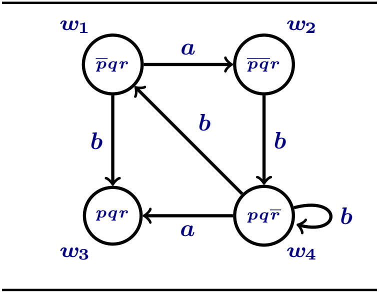

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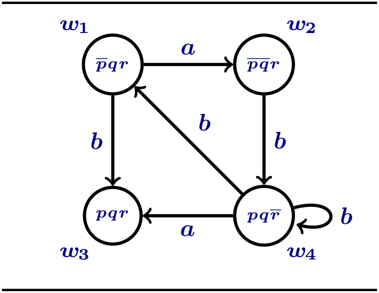

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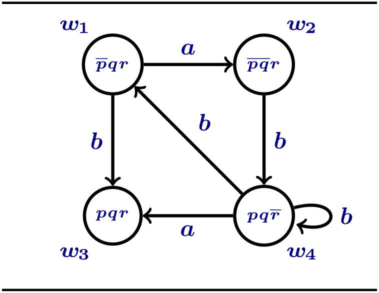

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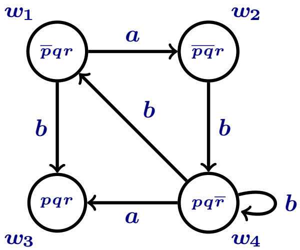

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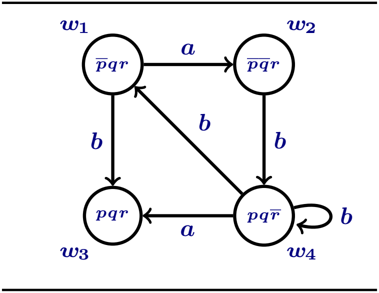

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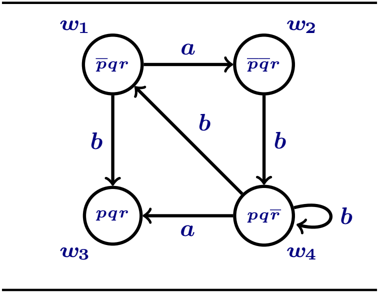

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- If we work only with models in which R is **symmetric**, then the following formula is valid:

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- If we work only with models in which R is **euclidean**, then the following formula, the **negative introspection** principle, is valid:

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A formula that can be derived by following these principles in a *finite* number of steps is called a **theorem**.

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Prove that $\varphi \longrightarrow \psi$ implies $\Box\varphi \longrightarrow \Box\psi$

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2.	$\Box(\varphi \longrightarrow \psi)$	Nec from step 1
3.	$\Box(\varphi \longrightarrow \psi) \longrightarrow (\Box\varphi \longrightarrow \Box\psi)$	Axiom 2
4.	$\Box\varphi \longrightarrow \Box\psi$	MP from steps 2 and 3

More systems

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$T := K + \textit{veridicality}\ (\Box\varphi \rightarrow \varphi)$

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$S5 := S4 + \varphi \rightarrow \Box\Diamond\varphi$

$S4 + \textit{negative introspection}\ (\neg\Box\varphi \rightarrow \Box\neg\Box\varphi)$

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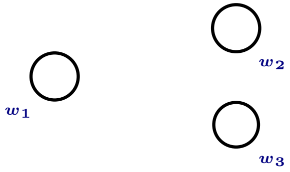

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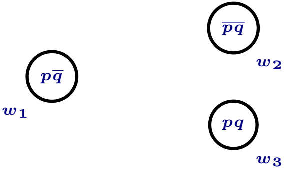

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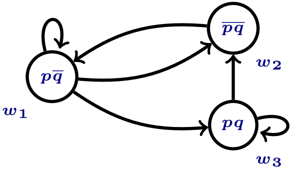

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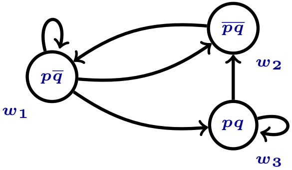

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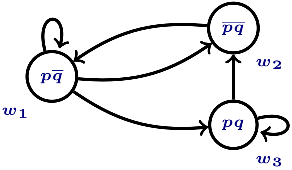

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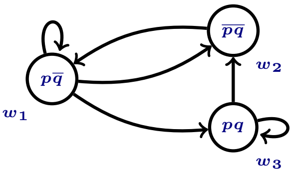

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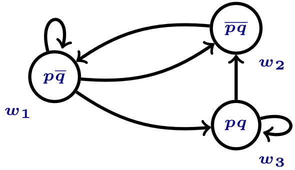

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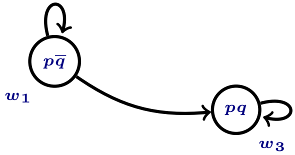

Formally,

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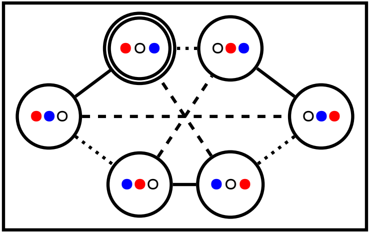

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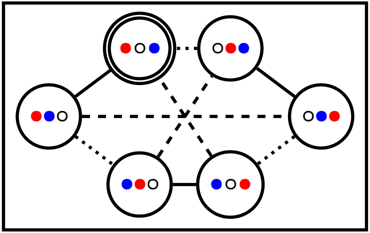

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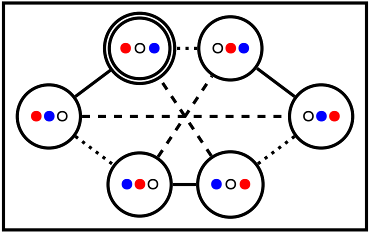

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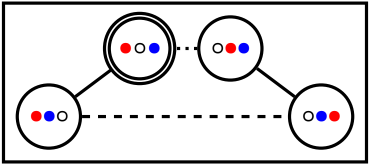

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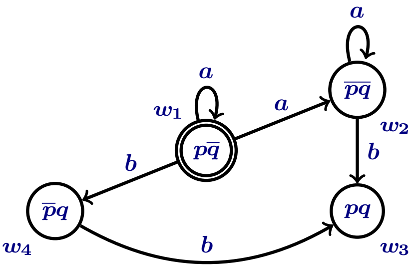

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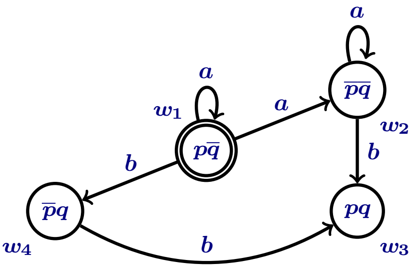

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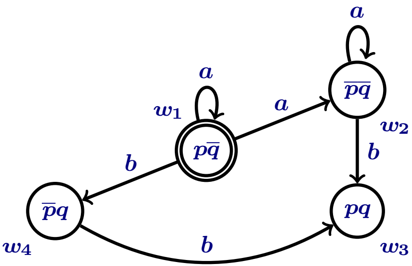

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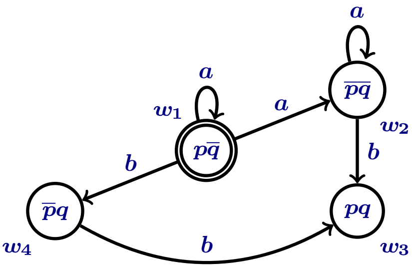

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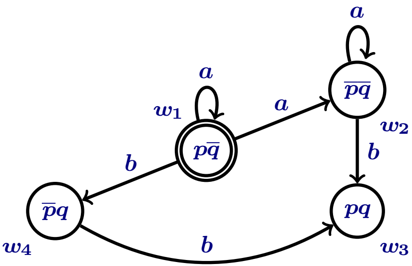

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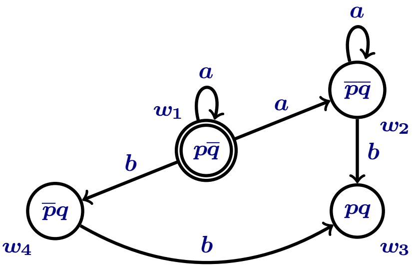

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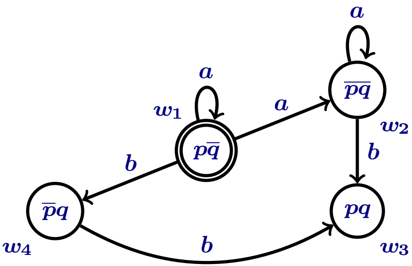

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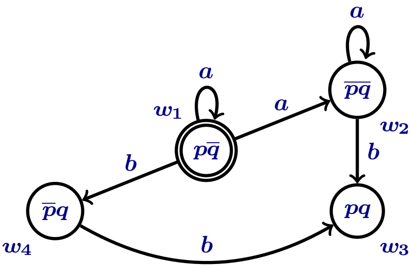

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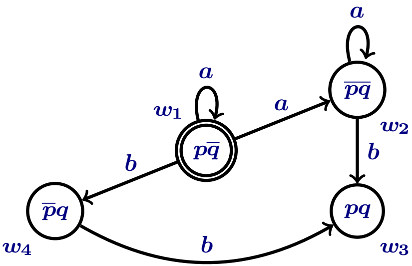

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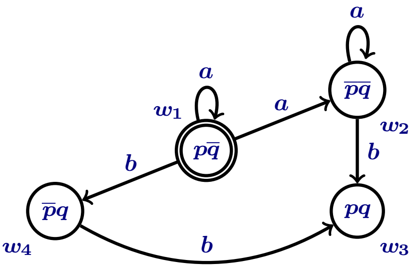

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