

# Functional Programming with Haskell, Part 3

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# Announcements

- ▶ Student hours this week
  - ▶ Thu 9/22 4-5pm, Ken, **remote only**
  - ▶ Fri 9/23 2:15-3:15pm, Bingyang, hybrid
- ▶ Final project presentations 12/14 6-9pm
  - ▶ Save the date!
- ▶ For next Wednesday:
  - ▶ HW1 due

# Today's Plan

- ▶ Functional Programming with Haskell

# Ranges

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# Ranges

- ▶ “Lists can also be given by not enumerating all their elements but by indicating the range of elements: `[n..m]` is the list bounded below by `n` and above by `m`.”
  - ▶ Works for objects that can be enumerated (i.e., converted to and from `Int`)
    - ▶ Integers
    - ▶ Characters
    - ▶ etc.
  - ▶ `[1..423]` is the list of all numbers from 1 to 423
  - ▶ `['g'..'s']` is the list of all characters from `g` to `s`

# Infinite Lists

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# Infinite Lists

- ▶ `[0..]` denotes the list of all natural numbers
- ▶ “Since Haskell does not evaluate an argument unless it needs it, it can handle infinite lists as long as it has to compute only a finite amount of its elements.”
  - ▶ `[0..]` does not terminate, but `take 5 [0..]` does



# Mapping

If you use the Hugs command `:t` to find the types of the function `map`, you get the following:

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map :: forall a b. (a -> b) -> [a] -> [b]
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The function `map` takes a function and a list and returns a list containing the results of applying the function to the individual list members.

If `f` is a function of type `a -> b` and `xs` is a list of type `[a]`, then `map f xs` will return a list of type `[b]`. E.g., `map (^2) [1..9]` will produce the list of squares

```
[1, 4, 9, 16, 25, 36, 49, 64, 81]
```

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- `(x op)` is the operation resulting from applying `op` to its lefthand side argument.
- `(op)` is the prefix version of the operator.
- Thus `(2^)` is the operation that computes powers of 2, and `map (2^) [1..10]` will yield  
`[2, 4, 8, 16, 32, 64, 128, 256, 512, 1024]`

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- `(op)` is the prefix version of the operator.
- Thus `(2^)` is the operation that computes powers of 2, and `map (2^) [1..10]` will yield  
`[2, 4, 8, 16, 32, 64, 128, 256, 512, 1024]`
- Similarly, `(>3)` denotes the property of being greater than 3, and `(<3)` the property of being smaller than 3.



# Map

If  $p$  is a property (an operation of type  $a \rightarrow \text{Bool}$ ) and  $l$  is a list of type  $[a]$ , then `map p l` will produce a list of type `Bool` (a list of truth values), like this:

```
Prelude> map (>3) [1..6]
[False, False, False, True, True, True]
Prelude>
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$$\text{map} :: (a \rightarrow b) \rightarrow [a] \rightarrow [b]$$

$$\text{map } f [] = []$$

$$\text{map } f (x:xs) = (f x) : \text{map } f xs$$

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filter p [] = []
```

```
filter p (x:xs) | p x      = x : filter p xs
                | otherwise =      filter p xs
```

# Guarded Equations

```
foo t | condition_1 = body_1  
      | condition_2 = body_2  
      | otherwise   = body_3
```

- ▶ If condition\_1 is true, then `foo t = body_1`
- ▶ Else if condition\_2 is true, then `foo t = body_2`
- ▶ Else, `foo t = body_3`



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- ▶ Can also be written as:

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foo t = if condition_1 then body_1
        else if condition_2 then body_2
        else body_3
```

- ▶ Guards are more common, though, especially when you have multiple if conditions

## List comprehension

List comprehension is defining lists by the following method:

```
[ x | x <- xs, property x ]
```

This defines the sublist of `xs` of all items satisfying `property`. It is equivalent to:

```
filter property xs
```

# Examples

```
someEvens      = [ x | x <- [1..1000], even x ]  
  
evensUntil n  = [ x | x <- [1..n], even x ]  
  
allEvens      = [ x | x <- [1..], even x ]
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someEvens      = [ x | x <- [1..1000], even x ]  
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```

Equivalently:

```
someEvens      = filter even [1..1000]  
evensUntil n  = filter even [1..n]  
allEvens      = filter even [1..]
```

# Function Composition

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- Haskell implementation:

$$\begin{aligned}
 (.) &:: (a \rightarrow b) \rightarrow (c \rightarrow a) \rightarrow (c \rightarrow b) \\
 f \cdot g &= \lambda x \rightarrow f (g x)
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- Note the types!

# Type Classes

- ▶ **Exercise 3.7** Check the type of the function  $(\backslash x y \rightarrow x \neq y)$  in Haskell. What do you expect? What do you get? Can you explain what you get?

# Type Classes

- ▶ **Exercise 3.7** Check the type of the function  $(\backslash x y \rightarrow x /= y)$  in Haskell. What do you expect? What do you get? Can you explain what you get?
- ▶  $(\backslash x y \rightarrow x /= y) :: \text{Eq } a \Rightarrow a \rightarrow a \rightarrow \text{Bool}$ 
  - ▶  $(\backslash x y \rightarrow x /= y)$  has type  $a \rightarrow a \rightarrow \text{Bool}$ , where  $a$  is in type class  $\text{Eq}$

# Type Classes

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  - ▶ Like Java interfaces, not like Java (or Python) classes

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- ▶ **Type classes** are collections of types that implement certain behaviors
  - ▶ Like Java interfaces, not like Java (or Python) classes
  - ▶ `Eq` contains types that can be compared for equality (i.e., that implement `(==)` and `(/=)`)
  - ▶ `Ord` contains types that can be ordered (i.e., that implement `(<=)` and `compare`)
  - ▶ `Enum` contains types that can be enumerated
  - ▶ `Show` contains types that can be printed (i.e., that can be presented as strings)
  - ▶ etc.

# Type Classes

- ▶ **Exercise 3.8** Is there a difference between  $(\backslash x y \rightarrow x /= y)$  and  $(/=)$ ?

# Type Classes

- ▶ **Exercise 3.8** Is there a difference between  $(\lambda x y \rightarrow x \neq y)$  and  $(\neq)$ ?
- ▶ **Eta reduction**: One can convert between  $\lambda x.f(x)$  and  $f$  whenever  $x$  does not appear free in  $f$



## elem, all, and

```
elem :: Eq a => a -> [a] -> Bool
elem x []      = False
elem x (y:ys) = x == y || elem x ys
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all :: Eq a => (a -> Bool) -> [a] -> Bool
all p = and . map p
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Note the use of `.` for function composition.

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Note the use of `.` for function composition.

```
and :: [Bool] -> Bool
and [] = True
and (x:xs) = x && and xs
```

## Sonnet 73

```

sonnet73 =
  "That time of year thou mayst in me behold\n"
  ++ "When yellow leaves, or none, or few, do hang\n"
  ++ "Upon those boughs which shake against the cold,\n"
  ++ "Bare ruin'd choirs, where late the sweet birds sang.\n"
  ++ "In me thou seest the twilight of such day\n"
  ++ "As after sunset fadeth in the west,\n"
  ++ "Which by and by black night doth take away,\n"
  ++ "Death's second self, that seals up all in rest.\n"
  ++ "In me thou see'st the glowing of such fire\n"
  ++ "That on the ashes of his youth doth lie,\n"
  ++ "As the death-bed whereon it must expire\n"
  ++ "Consumed with that which it was nourish'd by.\n"
  ++ "This thou perceivest, which makes thy love more strong,\n"
  ++ "To love that well which thou must leave ere long."

```



# Counting

```
count :: Eq a => a -> [a] -> Int
count x []           = 0
count x (y:ys) | x == y    = succ (count x ys)
               | otherwise = count x ys
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```
average :: [Int] -> Rational
average [] = error "empty list"
average xs = toRational (sum xs) / toRational (length xs)
```

# Nub

nub removes duplicates, as follows:

```
nub :: Eq a => [a] -> [a]
```

```
nub [] = []
```

```
nub (x:xs) = x : nub (filter (/= x) xs)
```



## Some Commands to Try Out

- `putStrLn sonnet73`
- `map toLower sonnet73`
- `map toUpper sonnet73`
- `filter ('elem' "aeiou") sonnet73`
- `count 't' sonnet73`
- `count 't' (map toLower sonnet73)`
- `count "thou" (words sonnet73)`
- `count "thou" (words (map toLower sonnet73))`

# Hello World!

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- ▶ `main` is the entry point to a compiled program
- ▶ `IO a` is the type of a function that performs an I/O action and returns an object of type `a` in a box
  - ▶ Printing a string doesn't really have a return value, so we return the empty tuple (i.e., `unit ()`)

# Computing with Boxes

- ▶ Why boxes?
  - ▶ Haskell functions are supposed to be **pure**
    - ▶ Do not change state
    - ▶ If you call a function twice with the same arguments, you should get the same results each time

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  - ▶ But I/O actions have **side effects**
    - ▶ Communicate with and change the state of the outside world
  - ▶ Boxes separate the pure and impure parts of our programs



# Computing with Boxes

```
f :: IO ()  
f = do  
    s <- getLine  
    putStrLn ("Hello " ++ s ++ "!")
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- ▶ Do syntax
  - ▶ “Glues” I/O actions together
- ▶ `<-` (pronounced **bind**) gets stuff out of boxes
  - ▶ `getLine :: IO String` waits for the user to input a string, and then puts it in a box
  - ▶ We then open the box and bind the contents to `s`

# String Processing

- ▶ What are the differences between the following functions?
  - ▶ `show`
  - ▶ `putStr`
  - ▶ `putStrLn`
  - ▶ `print`

# String Processing

- ▶ What are the differences between the following functions?
  - ▶ `show` takes an object of type `a`, where `a` is in type class `Show`, and presents it as a string
    - ▶ Quotes its argument, by putting double quotes around it
  - ▶ `putStr` takes an object of type `String`, and prints it (without quotes)
  - ▶ `putStrLn` is like `putStr`, except it also prints a newline character
  - ▶ `print` takes an object of type `a`, where `a` is in type class `Show`, and prints it as a string
    - ▶ Equivalent to `(putStrLn . show)`
    - ▶ Expressions input to the Haskell interpreter are implicitly printed

# File Processing

```
Prelude> :t readFile
readFile :: FilePath -> IO String
Prelude> :t writeFile
writeFile :: FilePath -> String -> IO ()
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- ▶ `readFile` takes a `FilePath` (i.e., `String`) and outputs an `IO` action that reads the file and puts its contents in a box
- ▶ `writeFile` and `appendFile` take a `FilePath` and a `String` and return an `IO` action that writes the string to the file
  - ▶ `writeFile` overwrites the file, while `appendFile` concatenates the string to the end