# Propositional Logic

Kenneth Lai

Brandeis University

September 28, 2022

# Announcements

- By 11:59pm today
  - HW1 due
- For next Monday
  - Read van Eijck and Unger Chapter 4.2, 5.6, 6.3, 7.5, 7.6
    - Look at `Model.hs` and `TCOM.hs`
- For 10/19
  - HW2 due
    - HW2 will be posted by next Monday
  - Paper Presentation Ideas due

# Paper Presentations

- In pairs or small groups, students will read and present a paper of their choice from the computational semantics literature
  - (i.e., groups of 2 or 3)
  - Sometime between 11/16 and 12/5
  - Groups should aim for around 20 minutes for summary and analysis, and around 5 minutes for questions and discussion

# Paper Presentations

- By 10/19, please prepare a short document (one per group, in PDF format) containing:
  - Names of group members
    - We can help you find a group if needed
  - 2 (or more) possible papers you would be willing to present

# Paper Presentations

- If you know what you want to want to present, great!
- If not, that's fine too
  - For the next few classes, we will take a few minutes at the beginning of class to discuss possible topics/example papers
  - Suggestions welcome!

# Paper Presentations

- General resources/places to look for papers
  - Conference proceedings
    - Specific to computational semantics: IWCS, *SEM
    - General CL/NLP: ACL, NAACL, EACL, AACL, COLING, LREC, etc. (ACL Anthology)
  - Workshop proceedings
    - Any workshop affiliated with any of the above (especially IWCS or *SEM)
  - Journals
    - General CL/NLP: Computational Linguistics, TACL, etc.

# Today's Plan

- Paper Presentation Idea: Computational Lexical Semantics
- Propositional Logic
  - Syntax
  - Semantics
- Predicate Logic
  - Syntax
  - Semantics

# Today's Plan

- Paper Presentation Idea: Computational Lexical Semantics
- Propositional Logic
  - Syntax
  - Semantics
- Predicate Logic
  - Syntax
  - Semantics
- (we'll see how far we get...)

# Computational Lexical Semantics

- A few foundational papers
  - FrameNet: Charles J. Fillmore, Christopher R. Johnson, and Miriam R.L. Petruck. 2003. Background to FrameNet. International Journal of Lexicography, 16(3):235–250.
  - PropBank: Martha Palmer, Daniel Gildea, and Paul Kingsbury. 2005. The Proposition Bank: An Annotated Corpus of Semantic Roles. Computational Linguistics, 31(1):71–106.
  - Generative Lexicon: James Pustejovsky. 1991. The Generative Lexicon. Computational Linguistics, 17(4):409–441.
  - WordNet: George A. Miller, Richard Beckwith, Christiane Fellbaum, Derek Gross, and Katherine J. Miller. 1990. Introduction to WordNet: An On-line Lexical Database. International Journal of Lexicography, 3(4):235–244.

# Computational Lexical Semantics

- More recent work
  - FrameNet: FrameNet Bibliography
  - PropBank: PropBank Bibliography
  - Generative Lexicon: International Conference on the Generative Lexicon
  - WordNet: Global WordNet Conference

# Summary of 9/14 Discussion

| Things in model | Expression | Type |
|---|---|---|
| relations | verbs | `String` |
| entities | nouns | `String` |
| ? | adjectives | `String` |
| ? | sentences | `String` |

# Summary of 9/14 Discussion

| Things in model | Expression | Type |
|---|---|---|
| relations | verbs | `String` |
| entities | nouns | `String` |
| ? | adjectives | `String` |
| ? | sentences | `String` |

- We want our model to contain entities and relations between them

# Summary of 9/14 Discussion

| Things in model | Expression | Type |
|---|---|---|
| relations | verbs | `String` |
| entities | nouns | `String` |
| ? | adjectives | `String` |
| truth values | sentences | `String` |

- We want our model to contain entities and relations between them
- Since we are interested in truth conditions, our model should also contain truth values
  - These correspond to (declarative) sentences

# Summary of 9/14 Discussion

| Things in model | Expression | Type |
|---|---|---|
| relations | verbs | `String` |
| entities | nouns | `String` |
| ? | adjectives | `String` |
| truth values | sentences | `String` |

- We want our model to contain entities and relations between them
- Since we are interested in truth conditions, our model should also contain truth values
  - These correspond to (declarative) sentences
- Propositional logic is the logic of truth values

# Summary of 9/14 Discussion

| Things in model | Expression | Type |
|---|---|---|
| relations | verbs | `String` |
| entities | nouns | `String` |
| ? | adjectives | `String` |
| truth values | sentences | `String` |

- We want our model to contain entities and relations between them
- Since we are interested in truth conditions, our model should also contain truth values
  - These correspond to (declarative) sentences
- Propositional logic is the logic of truth values
- Predicate (or first-order) logic is the logic of entities, relations (or predicates), and truth values

# Propositional Logic

- Atomic propositions
  - Typically indicated by lower case letters $p$, $q$, $r$, etc., possibly with indices
  - Represent the meanings of certain declarative sentences
    - Specifically, those that cannot be decomposed into other atomic propositions and logical connectives

# Propositional Logic

- Atomic propositions
  - Typically indicated by lower case letters $p$, $q$, $r$, etc., possibly with indices
  - Represent the meanings of certain declarative sentences
    - Specifically, those that cannot be decomposed into other atomic propositions and logical connectives
  - For example, let:
    - $p$ be "It rains"
    - $q$ be "The sun is shining"
    - $r$ be "There will be a rainbow"

# Propositional Logic

- Formulas of propositional logic
  - Atomic propositions are formulas

# Propositional Logic

- Formulas of propositional logic
  - Atomic propositions are formulas
  - Let $F_1$ and $F_2$ be formulas. Then the following are also formulas:
    - Negation: $\neg F_1$ ("not $F_1$")
    - Conjunction: $(F_1 \wedge F_2)$ ("$F_1$ and $F_2$")
    - Disjunction: $(F_1 \vee F_2)$ ("$F_1$ or $F_2$")
    - Implication (or conditional): $(F_1 \rightarrow F_2)$ ("if $F_1$ then $F_2$")
    - Equivalence (or biconditional): $(F_1 \leftrightarrow F_2)$ ("$F_1$ if and only if $F_2$")

# Propositional Logic

- Formulas of propositional logic
  - Atomic propositions are formulas
  - Let $F_1$ and $F_2$ be formulas. Then the following are also formulas:
    - Negation: $\neg F_1$ ("not $F_1$")
    - Conjunction: $(F_1 \wedge F_2)$ ("$F_1$ and $F_2$")
    - Disjunction: $(F_1 \vee F_2)$ ("$F_1$ or $F_2$")
    - Implication (or conditional): $(F_1 \rightarrow F_2)$ ("if $F_1$ then $F_2$")
    - Equivalence (or biconditional): $(F_1 \leftrightarrow F_2)$ ("$F_1$ if and only if $F_2$")
- For example, the sentence "If it rains and the sun is shining, then there will be a rainbow" can be represented as the propositional formula $(p \wedge q) \rightarrow r$

# Semantics of Propositional Logic

- Valuations
  - Functions from atomic propositions to truth values $\{0, 1\}$ (or $\{\mathsf{F}, \mathsf{T}\}$)
  - Equivalently, a valuation can be represented as the set of atomic propositions that are true (in some model)

# Semantics of Propositional Logic

- The truth of an atomic proposition in a model is determined by the valuation in the model
- For other formulas:
  - $\neg F_1$ is true iff $F_1$ is false
  - $F_1 \wedge F_2$ is true iff $F_1$ is true and $F_2$ is true
  - $F_1 \vee F_2$ is true iff $F_1$ is true or $F_2$ is true
  - $F_1 \rightarrow F_2$ is true iff $F_1$ is false or $F_2$ is true
  - $F_1 \leftrightarrow F_2$ is true iff $F_1$ and $F_2$ have the same truth value

# Semantics of Propositional Logic

Another way of presenting the semantics of the propositional connectives is by means of *truth tables*, which specify how the truth value of a complex formula is calculated from the truth values of its components.

| $F_1$ | $F_2$ | $\neg F_1$ | $F_1 \wedge F_2$ | $F_1 \vee F_2$ | $F_1 \rightarrow F_2$ | $F_1 \leftrightarrow F2$ |
|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 1 |

# Semantics of Propositional Logic

- A formula $F$ is:
  - a tautology iff it is true for any valuation
  - a contradiction iff it is false for any valuation
  - satisfiable iff it is true for at least one valuation
  - contingent iff if is satisfiable but not a tautology

# Semantics of Propositional Logic

- A formula $F$ is:
  - a tautology iff it is true for any valuation
  - a contradiction iff it is false for any valuation
  - satisfiable iff it is true for at least one valuation
  - contingent iff if is satisfiable but not a tautology
- Two formulas $F_1$ and $F_2$ are logically equivalent iff they have the same truth value for any valuation
  - $F_1 \equiv F_2$
- "Formulas $P_1, \ldots, P_n$ logically imply formula $C$ ($P$ for premise, $C$ for conclusion) if every valuation which makes every member of $P_1, \ldots, P_n$ true also makes $C$ true."
  - $P_1, \ldots, P_n \models C$

# Propositional Logic

- "The logical connectives $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$ were designed to correspond to the natural language connectives *not*, *and*, *or*, *if...then*, and *if and only if*."
  - This is not always exact, though

# Propositional Logic

- "The logical connectives $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$ were designed to correspond to the natural language connectives *not*, *and*, *or*, *if...then*, and *if and only if*."
  - This is not always exact, though
- **Exercise 4.9** Translate the following sentences into propositional logic, making sure that their truth conditions are captured. What shortcomings do you encounter?
  - The wizard polishes his wand and learns a new spell, or he is lazy.
  - The peasant will deal with the devil only if he has a plan to outwit him.
  - If neither unicorns nor dragons exist, then neither do goblins.

# Propositional Logic

- "The logical connectives $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$ were designed to correspond to the natural language connectives *not*, *and*, *or*, *if...then*, and *if and only if*."
  - This is not always exact, though
- **Exercise 4.9** Translate the following sentences into propositional logic, making sure that their truth conditions are captured. What shortcomings do you encounter?
  - The wizard polishes his wand and learns a new spell, or he is lazy.
  - The peasant will deal with the devil only if he has a plan to outwit him.
  - If neither unicorns nor dragons exist, then neither do goblins.
  - (Bonus:) If kangaroos had no tails, [then] they would topple over. (Lewis, 1973)

# Propositional Logic

- **Exercise 4.10** The logical connective $\vee$ is inclusive, i.e. $p \vee q$ is true also if both $p$ and $q$ are true. In natural language, however, *or* is usually used exclusively, as in
  - You can either have ice cream or candy floss, but not both.

  Define a connective $\oplus$ for exclusive *or*, using the already defined connectives.

# Propositional Logic

- **Exercise 4.10** The logical connective $\vee$ is inclusive, i.e. $p \vee q$ is true also if both $p$ and $q$ are true. In natural language, however, *or* is usually used exclusively, as in
  - You can either have ice cream or candy floss, but not both.

  Define a connective $\oplus$ for exclusive *or*, using the already defined connectives.

  - N.B.: Given only, e.g., $\neg$ and $\wedge$, or $\neg$ and $\vee$, it is possible to define each of the other connectives